Collusion in Markets with Syndication*

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Abstract

Many markets, including markets for IPOs and debt issuances, are syndicated: each winning bidder invites competitors to join its syndicate to complete production. Using repeated extensive form games, we show that collusion in syndicated markets may become *easier* as market concentration falls, and that market entry may *facilitate* collusion. In particular, firms can sustain collusion by refusing to syndicate with any firm that undercuts the collusive price (and thereby raising that firm's production costs). Our results can thus rationalize the paradoxical empirical observations that the IPO underwriting market exhibits seemingly collusive pricing despite low levels of market concentration.

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1 Introduction

The fees that investment banks collect for initial public offerings (IPOs) strongly suggest collusive behavior, with apparent coordination on fees equal to 7% of issuance proceeds for small- to moderate-sized IPOs (Chen and Ritter, 2000).[1] At the same time, the number of investment banks running moderately sized IPOs is quite large, and there appears to be a nontrivial amount of entry and exit in the market (Hansen, 2001); this presents a puzzle, as standard industrial organization intuitions would therefore suggest that pricing should be competitive. In this paper, we provide a pathway by which the structure of the IPO underwriting market—and other similarly organized markets—can make it possible to sustain high prices even in the presence of low market concentration.[2]

The market for running IPOs is syndicated; once an issuer contracts with an investment bank to underwrite its IPO, that investment bank then organizes a syndicate to complete the IPO. We show that the presence of syndication can reverse standard intuition regarding the effect of market concentration. Indeed, below a certain level of concentration, the scope for collusion in a syndicated market *increases* as concentration declines: Colluding firms can punish a firm that undercuts the collusive price by refusing to participate in that firm's syndicate; this type of in-period punishment becomes more powerful as the market becomes less concentrated, because there are greater returns to joint production when firms are smaller.

The collusive strategies we consider here just depend on the syndicated structure of the market; thus, our work also demonstrates the potential for collusion in other settings where syndication is prevalent. For instance, Officer et al. (2010) raised concerns about collusion among private equity firms working together on syndicated leveraged buyouts, and, in work motivated by ours, Cai et al. (2018) examined the scope for collusion in the syndicated lending

[1] The 7% spread has recently attracted the attention of Commissioner Robert J. Jackson Jr. of the Securities and Exchange Commission who, noting that "middle-market entrepreneurs still have to pay 7% of what they've created to access our public markets," has become concerned that this "IPO tax" discourages firms from going public (Jackson, 2018).

[2] The annual funding raised by firms through IPOs ranges from a few billion dollars to over $60 billion a year (Ritter, 2018); perhaps more importantly, an IPO is an essential step towards accessing the seasoned equity market and public bond markets.

market.[3] Outside of the finance industry, syndication through horizontal subcontracting is also common. Examples include construction, transportation, communications, and military procurement.[4] Moreover, antitrust authorities have noted that collusive behavior seems to be more common in industries in which horizontal subcontracting is prominent; this experience has led them to heighten scrutiny of those industries.[5]

We model a market with syndication as a repeated extensive form game. In each period, firms compete on price for the opportunity to complete a single project. Upon being selected, the chosen firm may invite additional firms to join in the production process, forming a "syndicate." Recruiting additional firms is valuable because each firm's production cost is convex in the amount of production assigned to that firm.[6] Each invited firm then decides whether to join the syndicate. Finally, the project is completed by the syndicate members, payoffs are realized, and play proceeds to the next period.

We show that in markets with syndication, less concentrated markets may have prices that are further from their marginal costs of production. In particular, the highest price that can be sustained under equilibrium play is a U-shaped function of market concentration: When markets are very concentrated, collusion can be sustained as in many standard industrial organization models: after a firm undercuts on price, all firms revert to a "competitive" equilibrium in which firms earn no profits in subsequent periods.[7] However, when firms are

[3] Private equity raised around \$230 billion (McKinsey & Co., 2018), with the specific syndicated (or club) LBO deals studied in Officer et al. (2010) alone peaking at \$113 billion in 2007, while syndicated lending, one of the most important mechanisms for firm borrowing, reached \$2.7 trillion in issuance in the United States in 2017 (Thompson Reuters, 2019).

[4] For a discussion of industries where horizontal subcontracting is important, see Spiegel (1993), Aronstein et al. (1998), Gil and Marion (2012), and Marion (2015).

[5] See the U.S. Antitrust Guidelines for Collaboration Among Competitors (Federal Trade Commission, 2000) and the Department of Justice primer on "Price Fixing, Bid Rigging, and Market Allocation Schemes: What They Are and What to Look For" (Department of Justice, 2015), as well as the EU Guidelines on the Applicability of Article 101 of the Treaty on the Functioning of the European Union to Horizontal Co-operation Agreements (European Commission, 2011).

[6] Convex production costs for underwriters of fixed size are a relatively standard assumption in the IPO literature; see Khanna et al. (2008) for a microfoundation for this assumption based on an inelastic supply of investment banking talent (and see also Lyandres et al. (2016) for further discussion). Corwin and Schultz (2005) provide a thorough discussion of the role of underwriting syndicates in the IPO process, with a particular emphasis on the role of the syndicate in information production through access to investors; such an information-based model is consistent with our assumed cost function.

[7] See, for instance, Tirole (1988).

small, completing the project alone is very costly, and thus collusion can be sustained by in-period punishments: after a firm undercuts on price, other firms can punish the undercutting firm in the same period by refusing to join its syndicate (and thus dramatically increasing the undercutting firm's costs of production). Of course, such behavior by other firms must itself be incentive compatible. Thus, firms that reject offers of syndication from a firm that undercuts on price must be rewarded in future periods; moreover, to induce firms to turn down more attractive syndication offers, those firms must be promised greater rewards in subsequent periods.[8]

In repeated normal form games, punishments can be enforced using the *simple penal codes* of Abreu (1986), under which only one punishment strategy is needed for each player, regardless of the timing or nature of the deviation. However, as noted by Mailath et al. (2017), in the analysis of repeated extensive form games it is necessary to consider more complex responses to deviations.[9,10] In particular, in our setting, it is key that firms punish a price undercutter *in-period* by refusing the undercutter's offers of syndication; to do this, we must construct a strategy profile that simultaneously punishes a firm that undercuts on price

[8]Collusive pricing in our setting is driven by the role of syndication in efficient production, rather than simply by our baseline model's assumption that industry capacity is fixed and thus each firm's productive capacity decreases as the number of firms increases. The assumption of fixed industry capacity reflects conditions of the market for IPOs (see Section 2.1). However, if each firm's productive capacity does not vary with the number of firms, prices still do not fall to the marginal cost of production, even as the number of firms grows large; indeed, markups may increase as the number of firms grows (see Section 3.4).

[9]It is not sufficient to consider the repeated version of the reduced normal form game, as the equilibria of that game will not necessarily correspond to subgame-perfect equilibria of the original repeated extensive form game.

[10]Nocke and White (2007) were the first to use the theory of repeated extensive form games to study collusion, showing that vertical mergers can facilitate collusion under certain circumstances. Byford and Gans (2014) consider collusion via market segmentation by considering a repeated extensive form game with market segment entry decisions followed by production decisions; however, they restrict attention to a class of equilibria in which agents' decisions regarding production can not depend on past play, eliminating the extensive-form considerations which are central to our work here. See also the work of Atakan and Ekmekci (2011), who consider how reputation may be built in a repeated extensive form game with initial uncertainty about one player's type. Subsequent to the current paper, Hatfield et al. (2019a) used a repeated extensive form game approach to evaluate the scope for collusion in brokered markets such as real estate agency.

and rewards firms that refuse to join a price undercutter's syndicate.[11,12]

Our baseline model considers the case of symmetric firms; we then extend our results to markets with heterogeneous firms. As in the case with symmetric firms, we find that heterogeneous firms can collude even when the market is very fragmented. Indeed, heterogeneity itself can increase firms' ability to collude. Moreover, the entry of small firms enhances the scope for collusion in markets with syndication, again counter to standard results in the theory of industrial organization.[13]

Whether spreads on IPOs are set in a competitive or collusive manner has been debated in the finance literature since Chen and Ritter (2000) first documented the clustering of IPO spreads at 7%.[14] Abrahamson et al. (2011) documented that the spreads for IPOs are significantly higher in the United States than in Europe, and cited this as evidence that pricing in the U.S. underwriting market is collusive; Lyandres et al. (2016) also provided empirical results consistent with implicit collusion. Kang and Lowery (2014) presented and estimated a formal model of why collusion would lead to the observed clustering on spreads, using insights on collusive behavior from Athey et al. (2004);[15] moreover, while spreads

[11]To our knowledge, we are the first to model syndication, i.e., subcontracting, in a repeated extensive form game. There is, however, a large literature on horizontal subcontracting in the context of one-shot interactions, starting with the work of Kamien et al. (1989); see also the work by, among others, Spiegel (1993) and Shy and Stenbacka (2003).

[12]Brock and Scheinkman (1985) consider an unrelated model of Bertrand competition with capacity constraints. In that model, the stage game is a normal form game in which firms announce prices, with the lowest-priced firm making sales to the limit of its capacity before the next-lowest-priced firm makes sales, and so on. The highest sustainable price may be non-monotonic in the number of firms, as adding a firm makes the stage-game Nash equilibrium outcome less profitable for all firms (in addition to the usual effect that adding a firm reduces the profits to each colluding firm by dividing the profits of collusion among one more participant). (We also note that Brock and Scheinkman (1985) restrict their analysis to punishment strategies that are stage-game Nash equilibria, which are not necessarily optimal.)

The Brock and Scheinkman (1985) model is fundamentally different from ours: In their model, firms only interact in one step, in which they compete, while in ours firms also collaborate through a second, post-pricing step via the syndication process. Because of the syndication process in our model, there always exists a zero-profit (subgame perfect) Nash reversion equilibrium of the stage game; thus, the fundamental driver of their result—that the highest sustainable price can be non-monotonic in industry concentration—is not present in our model.

[13]Both Rosenthal (1980) and Chen and Riordan (2008) also consider models in which entry can increase prices; however, entry can increase prices in their settings for very different reasons than those examined here.

[14]The Department of Justice has also raised concerns about whether IPO spreads are anticompetitive, leading to a multi-year investigation starting in the late 1990s (Wall Street Journal, 2001).

[15]Kang and Lowery's work also helps to explain why, under collusion, spreads may not change with IPO size or changes over time in the cost of performing an IPO.

are constant up to a threshold of approximately $100 million, they decline for the largest IPOs in a manner consistent with the model of Rotemberg and Saloner (1986). By contrast, Hansen (2001) claims that the clustering of IPO spreads is likely to be the result of efficient contracting, documenting the apparent relative ease of entry and lack of concentration in the market. Moreover, Torstila (2003) documents the clustering of spreads at lower levels in countries other than the United States, arguing that this provides evidence that clustering does not imply collusive behavior. Our work can help reconcile the apparently conflicting evidence: we show that collusion in IPO markets is possible despite—and in fact may be facilitated by—low levels of market concentration.

There also is a related debate over whether the pricing of the IPO securities themselves is collusive. IPO shares generally gain about 15% on their first day of public trading, suggesting that issuers are "leaving money on the table" (Loughran and Ritter, 2004). Some authors argue that underpricing is a means for underwriters to extract rents from issuers—likely a feature of an uncompetitive market (Biais et al., 2002; Cliff and Denis, 2004; Loughran and Ritter, 2004; Liu and Ritter, 2011; Kang and Lowery, 2014). On the other hand, some argue that issuers may desire underpricing, and thus underpricing can occur even when underwriters compete aggressively (Rock, 1986; Allen and Faulhaber, 1989; Benveniste and Spindt, 1989; Chemmanur, 1993; Brennan and Franks, 1997; Stoughton and Zechner, 1998; Lowry and Shu, 2002; Smart and Zutter, 2003). While our work does not address the issue of underpricing directly, it does show that underwriters could collude in the market for IPOs, even though—or even because—the market is highly fragmented.

Beyond IPOs, many other financial markets are syndicated, including the markets for debt, reinsurance, and private equity. Motivated by our work, Cai et al. (2018) investigated the effects of market concentration on interest rates (i.e., prices) in the syndicated loan market; consistent with our theory, they found robust evidence that prices are indeed U-shaped in market concentration.

Meanwhile, numerous lawsuits have alleged that private equity firms used strategies similar to those that we describe here to support collusion;[16] these lawsuits[17] claim that firms "monitored compliance through... detailed 'scorecards' that listed the deals they worked on, who else was involved in those deals, and the resulting favors that they owed others and that others owed them."[18] Indeed, Officer et al. (2010) found that shareholders in firms bought out through leveraged buyouts (LBOs) received approximately 40% lower premiums in club deals (i.e., syndicated LBOs) compared to sole-sponsored LBOs. The plaintiffs observed that "KKR bragged to its investors in 2005: 'Gone are the days when buy-out firms fought each other with the ferocity of cornered cats to win a deal.'" Moreover, the plaintiffs argued that "Every time a Defendant's club signaled that it had a proprietary deal ... the other Defendants refused to submit a better offer—even when ... this enabled [an acquiring] club to purchase [a target] at such a low price, it amounted to [in the words of one Defendant] 'highway robbery.'" Plaintiffs offered evidence that defendants refused to work with outsiders (i.e., potential entrants and spoilers of collusion) who wanted to challenge allegedly collusive deals. The plaintiffs also argued that in exchange "for not competing for large LBOs," defendants were "offered an invitation to participate in that LBO" or a future LBO "with its co-conspirators" explicitly "as a reward." Indeed, firms "invited into a current deal understood that they were required to invite their co-conspirators into a subsequent deal."[19,20]

There have also been a number of instances of collusive bidding behavior facilitated by *ex post* horizontal subcontracting. For example, a group of companies and individuals were convicted of criminal antitrust violations for conspiring to rig the bidding for highway construction contracts in New York; one of the ways they maintained this conspiracy was

[16]This behavior was also the subject of a multi-year criminal investigation by the Department of Justice (Rosener and Mainali, 2013).

[17]All of the quotes in this paragraph are taken from the Fifth Amended Complaint in *Dahl et al. v. Bain Capital Partners, LLC, et al.*, 937 F. Supp. 2d 119 (D. Mass. 2013).

[18]For example, in one "instance, when Apollo co-founder Leon Black expressed his anger at Goldman Sachs' 'lack of reciprocity' for two deals he had invited Goldman Sachs to join, Goldman Sachs executives reviewed their scorecard and readily agreed that they 'truly need[ed] to involve [Apollo] soon in a principal deal [via syndication].'"

[19]These punishments clearly rely on the syndicated nature of the industry and play a similar role to the punishments we describe in our model.

[20]Following seven years of litigation, defendants settled the aforementioned lawsuit for approximately $600 million (Dezember, 2014).

by having winning bidders award lucrative subcontracts to the losing bidders. For instance, one winning bidder subcontracted construction of one mile of a thirteen-mile highway to a competitor at a price that was almost double the competitor's usual bid for the subcontract work.[21] In another example, firms performing concrete work on various nuclear facilities conspired to rig bids, and to compensate the non-winning bidders via subcontracts for work and materials.[22] Finally, the firms bidding on construction of Cairo's sewage system engaged in similar conduct, leading to multiple criminal and civil suits.[23]

The remainder of the paper is organized as follows: Section 2 introduces our model of markets with syndicated production; Section 3 characterizes the highest price sustainable via collusion in such markets. Section 4 considers how the highest sustainable price depends on market conditions. Section 5 extends the model to allow for contracting over production shares. Section 6 examines the impact of firm heterogeneity and market entry on the highest sustainable price. Section 7 concludes. Proofs and extensions of our main results are presented in Appendices A–D.

2 Model

We introduce a model of price competition in markets with syndication. There is a finite set of long-lived identical *firms* F and an infinite sequence of short-lived identical *buyers* $\{b_t\}_{t\in\mathbb{N}}$; we let $\varphi \equiv \frac{1}{|F|}$ be the *market concentration*. Time is discrete and infinite; firms discount the future at the rate $\delta \in (0, 1)$.

Each firm f is endowed with a production technology with a *cost function* $c(s, m)$, where s is the quantity of production done by firm f and m is the mass of the productive capacity controlled by firm f. We assume that the cost function is strictly increasing and strictly convex in the production done by the firm and strictly decreasing and strictly convex in

[21] See *New York v. Hendrickson Brothers, Inc.*, 840 F.2d 1065 (2d Cir. 1988).

[22] See *United States v. Inryco, Inc.*, 642 F.2d 290 (9th Cir. 1981).

[23] See *Elemary v. Holzmann*, 533 F. Supp. 2d 116 (D.C. 2008), *United States v. Anderson*, 326 F.3d 1319 (11th Cir. 2003), and *Miller v. Holzmann*, 563 F. Supp. 2d 54 (D.C. 2008). The cases concerned U.S. aid to Egypt used for the construction of a sewage system there.

the productive capacity of the firm.[24] We also assume that a firm which does not engage in production incurs no costs, i.e., $c(0, m) = 0$ for all m, and that production becomes arbitrarily costly as the productive capacity of the firm goes to 0, i.e., $\lim_{m \to 0} c(s, m) \geq \infty$ for all $s > 0$. Finally, we assume that the cost function is homogeneous of degree 1.[25]

We normalize the *total productive capacity* in the economy at $k = 1$;[26] in this section, we assume that the total productive capacity is evenly divided among the firms, so that the cost of producing s for any one firm is $c(s, \varphi)$.

In each period t, the firms and the buyer b_t play the following extensive-form stage game:

Step 1: Each firm $f \in F$ simultaneously makes a *price offer* $p_t^f \in [0, \infty)$. All offers to the buyer are immediately and publicly observed.[27]

Step 2: The buyer accepts at most one offer; this action is immediately and publicly observed. If no offer is accepted, the stage game ends.

Step 3: If the offer from some firm is accepted, then that firm becomes the *syndicate leader*, ℓ. Firm ℓ then simultaneously[28] offers each *non-leader firm* $g \in F \setminus \{\ell\}$ a *fee* w_t^g.[29] These offers are immediately and publicly observed.[30]

[24] This cost function assumption is equivalent to assuming that the industry consists of production units, each with convex production costs, with the production units divided evenly among firms.

[25] This last assumption is stronger than is generally necessary for our analysis but it greatly simplifies our presentation here. It is enough for our results that, as we proportionately increase the production required and the productive capacity, the cost function increases at a slower rate, i.e.,

$$\frac{\partial^2 c(s, sm)}{\partial s^2} \leq 0$$

for all $s, m > 0$; in the homogeneous case, this expression holds with equality. Economically, this implies that larger firms are weakly more efficient, in the sense that one firm with productive capacity sm can complete a production share s at a (weakly) lower cost than multiple firms with combined productive capacity sm.

[26] We consider the effects of changing the amount of productive capacity k in Section 4.

[27] Our analysis would be unchanged if we instead assumed that only the winning bid and bidder was publicly observed, as the strategies we construct to support the highest sustainable price given in Theorem 1 do not depend on the bids of the non-winning bidders. Moreover, if firms have access to a public randomization device, it is not necessary for our analysis that the winning bid be observable.

[28] The assumption of simultaneous offers simplifies the analysis; our results, however, are robust to an alternative specification of the game in which the syndicate leader makes sequential offers, which are accepted or rejected as they are received.

[29] In Section 5, we consider a "complete contracting" version of the model in which a syndicate offer specifies a firm's production share as well as its fee.

[30] An alternative setting in which syndication offers are not publicly observable is studied in further work

Step 4: Each firm $g \in F \smallsetminus \{\ell\}$ either accepts or rejects the fee w_t^g from ℓ. We call the set of firms that accept ℓ's offer, along with the firm ℓ, the *syndicate* G_t. At the end of the period, all firms observe the syndicate.[31]

The buyer b_t has a fixed *value* of $v > c(1,1)$ for the finished product.[32] Thus, the payoff to the buyer b_t is $v - p_t^f$ if he accepts the price offer from firm f and 0 if he does not accept any offer.

If the buyer b_t does not accept any offer, then each firm $f \in F$ obtains a payoff of 0. If firm ℓ becomes the syndicate leader (i.e., the buyer b_t accepts the offer of firm ℓ), then production is performed efficiently *ex post* by the members of ℓ's syndicate, and so each member of the syndicate performs an equal share of production. Thus, the stage game payoffs for the firms after a successful offer to the buyer from firm ℓ are as follows:

1. The payoff for ℓ is $p_t^\ell - c\left(\frac{1}{|G_t|}, \varphi\right) - \sum_{g \in G_t \smallsetminus \{\ell\}} w_t^g$, i.e., the price paid by the buyer less both the cost of ℓ's production and the fees paid to other firms.

2. The payoff for $g \in G_t \smallsetminus \{\ell\}$ is $w_t^g - c\left(\frac{1}{|G_t|}, \varphi\right)$, i.e., the fee paid to g less the cost of g's production.

3. The payoff for $h \in F \smallsetminus G_t$ is 0.

2.1 The IPO Industry Interpretation of the Model

It may be helpful to ground our model in a real-world example to illustrate the economic content of our assumptions. Consider the IPO process: After a would-be issuer decides to have an IPO, underwriting firms compete in a "bake-off" to be the lead underwriter,[33] or syndicate leader, for the IPO in a process analogous to the bidding in Steps 1 and 2 of our

(Hatfield et al., 2019b).

[31] Consequently, all firms know which syndication offers were accepted.

[32] We assume that $v > c(1,1)$ to avoid the trivial case where no trade is efficient.

[33] In some cases, large IPOs may in fact have multiple lead underwriters who may be selected at the bake-off stage.

game. The winning spread for a given IPO is publicly observable and is easily available to market participants (e.g., in the commonly used Securities Data Company (SDC) database).

After being selected, the lead underwriter recruits other banks—many or all of which may have competed to be lead underwriter—to help place the IPO shares, similar to Steps 3 and 4 of our game.[34] Each underwriter—both the lead underwriter and other syndicate members—"places" shares with important investors with which it has an ongoing relationship.[35] The placement process corresponds to the production process in our model; each underwriter's "book" of investors corresponds to that firm's productive capacity in our model. As investors are risk-averse and want diversified portfolios, it is more costly for an underwriter to place a large number of shares of one issuer with its set of investors than for a large set of underwriters to distribute those shares among all of their investors; this corresponds to our assumption that costs are convex in the amount of production performed. Similarly, a well-functioning syndicate, operating with access to the investor books of all of its members, would be expected to have the same ability to place shares with investors as a single firm operating with the same combined book of business; this matches our assumption that the cost function is homogeneous of degree 1. Total productive capacity corresponds to the set of institutional investors; this resembles our model, in which total productive capacity is unchanging with market structure.[36] However, in Section 6, we consider the possibility of entry by a new investment bank that brings new productive capacity to the market. We find that the effect of an entrant depends on that entrant's size: a sufficiently small entrant increases the scope for collusion, while a sufficiently large entrant may decrease it.

[34] The SDC database documents 4,576 U.S. IPOs conducted between 1970 and 2014 in which the issuer sold between \$20 and \$100 million worth of stock. Of these, 4,438—97%—were syndicated.

[35] Syndicate membership is also registered publicly and may be found in the SDC database; the syndicate may also be listed on the IPO "tombstone" used to advertise and commemorate the IPO.

[36] To more completely capture our institutional setting, we could extend the model to include a single time-0 "investor recruitment stage" in which firms compete to form links with institutional investors. It is straightforward to show that the extended game has an equilibrium which produces symmetric, collusive link formation and delivers the same price and the same profits as the equilibrium (of the original game) that provides the highest sustainable price given in Theorem 1.

3 Optimal Collusion

We now characterize the highest price sustainable via collusion in markets with syndication. A price p is *sustainable* if there exists a subgame perfect Nash equilibrium in which, along the equilibrium path, the buyer accepts a price offer of p in every period.

When the market is very concentrated, i.e., there are a small number of firms, any price (less than or equal to v) can be sustained by "grim trigger" strategies in which deviations from the collusive price are punished in subsequent periods by play in which every firm obtains 0 profits. This type of equilibrium is standard in the analysis of markets under Bertrand competition; in such markets, however, once there are enough firms in the market, no price above the cost of production can be sustained.

In markets with syndication, as in Bertrand competition markets, grim trigger strategies lose their bite as the number of firms in the market grows. However, unlike in the standard Bertrand competition model, markets with syndication admit a second way to maintain collusion: if a firm becomes a *price deviator*—i.e., if a firm bids lower than the price mandated by the collusive equilibrium—other firms can punish that firm "in period" by refusing offers of syndication. This raises the cost of production for that firm, as it must now complete the project on its own instead of engaging in (more efficient) syndicated production. To incentivize firms not to join the price deviator's syndicate, we need to promise them rewards in future periods; reverting to "perfect competition" after a price deviation does not accomplish this goal, as that would make all firms earn 0 profits in all future periods. Thus, reverting to perfect competition in periods after a price deviation is not the best continuation plan to sustain collusion in our setting. Instead, an optimal continuation plan should simultaneously reward firms for refusing offers of syndication while punishing the price deviator. In particular, the higher the price deviator's syndication offer to a firm g, the higher the continuation payoff needed to induce g to reject the offer of syndication; "the reward should fit the temptation," as highlighted by Mailath et al. (2017). It is also important to punish a firm if it accepts an offer of syndication from the deviating firm. We *do* revert to perfect competition if any firm accepts a price deviator's offer of syndication; this punishes *both* the initial deviator and any firm which joins the syndicate as harshly as possible. The strategies just described make recruiting a syndicate sufficiently costly that lone production is a more attractive option than recruiting a syndicate.[37]

Unlike grim trigger strategies, syndicate punishment strategies become *more* powerful as the market becomes less concentrated, as completing the project alone becomes increasingly expensive. Consequently, the preceding observations imply that, in general, the highest sustainable price is *not* monotone in market concentration: At high levels of market concentration, firms can collude at the monopoly price, as in the standard Bertrand competition model. When market concentration is sufficiently low, syndicate punishments again enable firms to collude at the monopoly price. However, at intermediate levels of market concentration, there are no subgame-perfect Nash equilibrium strategies that sustain the monopoly price.[38]

Our main result characterizes the highest sustainable price.

Theorem 1. *For $\delta \geq \frac{1}{2}$, the highest sustainable price, $p^\star$, is given by*

$$p^\star = \begin{cases} v & \varphi \in [1-\delta, 1] \\ \min\left\{\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}, v\right\} & \varphi \in (0, 1-\delta). \end{cases}$$

Moreover, either $p^\star$ is strictly single-troughed (and thus non-monotonic) in φ or $p^\star = v$ everywhere. Finally, $\lim_{\varphi \to 0} p^\star = v$.[39]

Figure 1 plots the highest sustainable price $p^\star$ as a function of φ. We call an equilibrium

[37] Cai et al. (2018) noted that there is anecdotal evidence for syndication "blacklists" in the syndicated loan market, which suggests that the types of punishment strategies we construct here are employed in practice.

[38] The syndication structure is essential for our result that pricing is non-monotonic in market concentration; a model in which the buyer directly contracts with individual firms to complete parts of a larger project exhibits pricing that is (weakly) decreasing in the number of firms. Thus, our non-monotonicity result is not driven exclusively by the fact that firm capacity is decreasing as the number of firms increases: see Section 3.4.

[39] We could also consider non-stationary equilibria, i.e., equilibria in which the prices offered to the buyers vary with time. However, in any such equilibrium, the offered price will never be above $p^\star$.

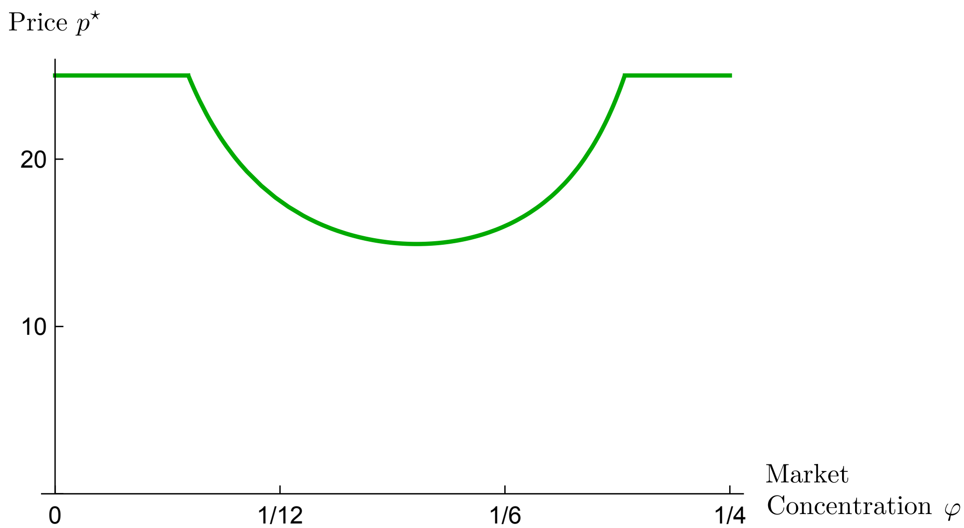

Figure 1: The highest sustainable price $p^\star$ as a function of market concentration φ. Here, $c(s,m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, and the maximum price that the buyer is willing to pay is $v = 25$. For sufficiently concentrated industries, the monopoly price can be sustained through grim trigger strategies. The highest sustainable price is lower for intermediate industry concentration levels, but as market concentration goes to 0 the highest sustainable price reaches the buyer's value v. The cost of efficient production (i.e., when the syndicate includes all firms) is 1 for all market concentrations φ.

in which, along the equilibrium path, the buyer accepts an offer of the highest sustainable price $p^\star$ and firms engage in efficient joint production a *maximally collusive equilibrium*. In a maximally collusive equilibrium, the combined per-period profits for all firms are given by $p^\star - c(1,1)$. A maximally collusive equilibrium maximizes industry profits; the buyer accepts the highest sustainable price, and efficient joint production ensures that costs are as low as possible.

We now describe an equilibrium that achieves the highest sustainable price $p^\star$ stated in Theorem 1; we give full details of the construction in Appendix A. Our equilibrium relies on two different punishment phases: First, we introduce a collusive punishment phase to incentivize firms not to join the syndicate of a price deviator, thus ensuring that no firm deviates on price. Meanwhile, we have a traditional Bertrand reversion phase, which is used to enforce prescribed behavior in the syndicate formation stage, both on and off the equilibrium path.

3.1 Bertrand Reversion Nash Equilibrium

We first describe the *Bertrand reversion Nash equilibrium* of the stage game, i.e., the subgame-perfect equilibrium in which all firms make zero profits and the buyer obtains the good at the lowest possible cost of production. In this equilibrium, each firm offers a price equal to the cost of producing the good when every firm participates in the syndicate. After the buyer chooses a firm to be the syndicate leader, that firm offers each non-leading firm a fee equal to its cost of production (assuming all syndication offers are accepted); each non-leading firm accepts this offer. Under the behavior just described, each firm in the syndicate other than the syndicate leader breaks even. Moreover, the syndicate leader also breaks even because the buyer pays exactly the cost of efficient production, the syndicate leader then pays each syndicate member exactly that member's production costs, and thus the payment retained by the syndicate leader is exactly its own cost of production.

Our first result shows that an equilibrium of the form just described exists and delivers each firm its lowest individually rational payoff.

Proposition 1. *There exists a subgame-perfect Nash equilibrium of the stage game, i.e., the Bertrand reversion Nash equilibrium, in which each firm obtains a payoff of* 0*, its lowest individually rational payoff.*

In standard repeated normal form games, reverting to the stage game equilibrium described in Proposition 1 would be sufficient to punish any off-equilibrium behavior. That is, the Bertrand reversion Nash equilibrium can be used to implement the simple penal codes of Abreu (1986). However, as noted by Mailath et al. (2017), simple penal codes are insufficient to characterize the equilibrium payoffs in repeated extensive form games. Even so, the Bertrand reversion Nash equilibrium is a key component of the equilibrium we use to support the highest sustainable price.

3.2 Maintaining Collusion with Grim Trigger Strategies When the Market Is Concentrated

We first show that the monopoly price v is sustainable when firms are patient and the number of firms is sufficiently small. Moreover, collusion can then be sustained via "grim trigger" strategies: after a deviation in either step of the stage game, play in all subsequent periods reverts to the Bertrand reversion Nash equilibrium described in Section 3.1.

Proposition 2. *If the discount factor is sufficiently high, i.e., $\delta \geq 1 - \varphi$, then there exists a subgame-perfect Nash equilibrium in which each firm offers the monopoly price in every period, i.e., $p_t^f = v$ for any $v \geq c(1,1)$, for all $f \in F$ and for all t.*

To prove Proposition 2, we construct an equilibrium in which each firm bids the monopoly price v in every period; each buyer then accepts one such offer (choosing each offer with equal probability). If the offer from firm ℓ is accepted, ℓ offers each non-leading firm a fee equal to its cost of production and each non-leading firm accepts this offer. If a firm offers a lower price in the first step, i.e., becomes a *price deviator*, the buyer chooses this lower price offer and syndication proceeds as if there were no price deviation; play then reverts to the Bertrand reversion Nash equilibrium described in Section 3.1 for subsequent periods.

Thus, in each period, the syndicate leader has profits of $v - c(1,1)$ while every other firm has 0 profits; hence, the expected discounted profits for a firm from following its equilibrium strategy is

$$\sum_{t=0}^{\infty} \delta^t \varphi(v - c(1,1)) = \frac{\varphi}{1-\delta}(v - c(1,1)).$$

Meanwhile, the upper bound on profits from offering an infinitesimally lower price is

$$v - c(1,1).$$

Thus, we can maintain monopoly prices using grim trigger strategies so long as $\delta > 1 - \varphi$. Proposition 2 is our setting's analogue to the familiar result in models of Bertrand competition that collusion at any price can be maintained by grim trigger strategies when the industry is sufficiently concentrated. However, in the standard model of Bertrand competition, collusion cannot be maintained at any price when $\delta < 1-\varphi$; in the next section, we show that collusion *can* be maintained in our setting for any $\delta \geq \frac{1}{2}$.

3.3 Maintaining Collusion with Syndicate Punishments

In this section, we first provide an intuitive description of an equilibrium that sustains the price $p^\star$ defined in Theorem 1 and then explain why no subgame-perfect Nash equilibrium can sustain a price higher than $p^\star$. The key idea is to construct strategies that exploit syndicate boycotting to enforce higher prices. Play begins in the *cooperation phase*, in which each firm offers the same price $p^\star$ and a firm, upon having its offer accepted, engages in efficient syndication. Play continues in the cooperation phase so long as no one deviates. If some firm deviates in the first step—i.e., offers a lower price to the buyer in order to guarantee that it wins the bid—we call such a firm a *price deviator*. Because of the efficiency gains from syndicated production, the price deviator will wish to induce the non-leading firms to join its syndicate, and thus will be willing to offer each firm a fee above its cost of production as an inducement. By the same token, if the non-leading firms refuse to join the price deviator's coalition, that will raise the deviator's cost of production, punishing the price deviator in-period, which discourages firms from deviating on price in the first place. Thus, the optimal collusive plan promises future-period rewards to non-leading firms that reject above-cost syndication offers from the price deviator; for this reason, Bertrand reversion after a price deviation is not necessarily the best continuation plan to sustain collusion. Moreover, to make rejecting the price deviator's syndication offer as attractive as possible, it is also important to punish a firm if it joins a price deviator's syndicate. To do so, we *do* use Bertrand reversion, as it punishes both the initial deviator and any firm that joins the syndicate as harshly as possible. Thus, whenever any firm deviates by accepting a price deviator's syndication offer—or rejecting a non-price deviator's equilibrium syndication offer—play enters the *Bertrand reversion phase*, in which firms play the Bertrand reversion Nash equilibrium each period.

After a period in which a firm f is a price deviator, but no firm joins its syndicate, play enters a *collusive punishment phase with continuation values* ψ which both punishes the price deviator f and rewards those firms that refused to join f's syndicate. In the collusive punishment phase, each firm offers the same *collusive punishment price* q to the buyer. The higher that q is, the higher total industry profits will be, permitting larger rewards to firms that rejected f's syndication offers. At the same time, behavior during a collusive punishment phase must itself be subgame-perfect; this imposes a constraint on how high q can be.[40] If each firm offers collusive punishment price q, then the winning bidder efficiently syndicates production; in so doing, it offers each non-leading firm g a fee equal to its assigned continuation value, ψ^g, plus g's production cost, $c(\varphi, \varphi)$. Finally, if any firm deviates from equilibrium play with respect to accepting or rejecting offers of syndication, play enters the *Bertrand reversion phase*, in which firms play the Bertrand reversion Nash equilibrium each period.

The continuation payoff ψ^g to a firm g other than the price deviator during a collusive punishment phase depends on the syndication offer made to g by the price deviator f. In particular, "the reward should fit the temptation" (Mailath et al., 2017)—the larger the fee offered to the firm by the price deviator, the greater the continuation payoff offered to that firm to induce it to reject the offer of syndication.[41] By making future play conditional on offers of syndication, firms are incentivized to punish price deviators in-period by refusing to join their syndicates—the more f offers g to join the syndicate, the more g receives in future periods; this mechanism deters non-deviators from accepting any syndication offer that the price-deviator would rationally make. That, in turn, reduces the incentive for firms

[40] If the price q is too high, then f or another firm will wish to price-deviate in this phase, and so the collusive punishment phase will not be subgame-perfect.

[41] The naked exclusion game of Section 3.2 of Mailath et al. (2017) is a simple example of a repeated extensive form game for which fitting the reward to the temptation lowers the discount factor necessary to sustain certain outcomes (relative to using "grim trigger"-like strategies).

to deviate on price, since each firm is aware that if it does so it will have to engage in lone production. Thus, the threshold for the highest sustainable price is set by the constraint that no firm can profitably deviate on price and engage in lone production. Since lone production becomes costlier as the market becomes more fragmented, reducing market concentration may make it easier to sustain collusion at a given price.

More formally, there are three phases of equilibrium play:

1. In the *cooperation phase*,
   - every firm submits the same bid $p = p^{\star}$,
   - the short-lived buyer accepts one such offer of $p^{\star}$, choosing each offer with equal probability,
   - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi)$ to every non-leading firm $g \in F \setminus \{\ell\}$ to join the syndicate, and
   - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
2. In the *collusive punishment phase with continuation values* ψ,
   - every firm submits the same bid $q = \min\{c(1, \varphi), v\}$,
   - the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,
   - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi) + \psi^{g}$ to every non-leading firm $g \in F \setminus \{\ell\}$ to join the syndicate, and
   - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
3. In the *Bertrand reversion phase*, firms play the Bertrand reversion Nash equilibrium.

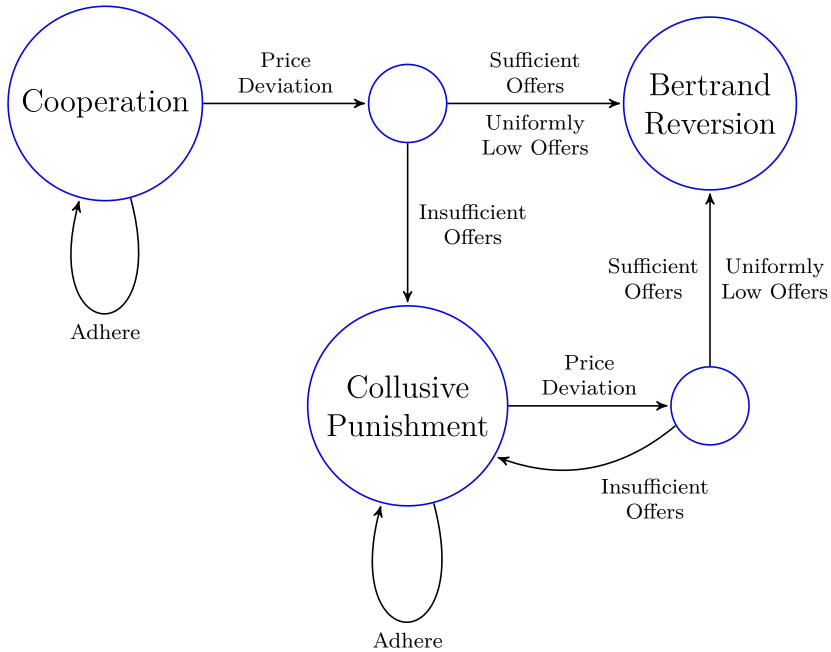

Figure 2: Automaton representation of the class of equilibria we consider. Labeled nodes are phases; unlabeled nodes are intermediate phases, which represent the branching of transitions based on behavior in the second step of the game.

We now describe how play transitions between equilibrium phases; the full phase diagram is pictured in Figure 2. In the cooperation and collusive punishment phases, so long as players adhere to their prescribed equilibrium strategies, play continues in the cooperation phase. If some firm f offers a lower price in the cooperation or a collusive punishment phase, or makes syndication offers inconsistent with equilibrium play, then future play depends on the syndication offers that are made:

- If the price deviator's syndication offers are so low that every firm expects to lose money this period were it to accept its syndication offer, then each firm rejects its syndication offer and we proceed to the Bertrand reversion phase (regardless of which syndication offers are accepted); we call this case *uniformly low offers*.

- If the price deviator's syndication offers are above the cost of production but are still not too high, then play proceeds to a collusive punishment phase if and only if every firm rejects the price deviator's syndication offers—how much each firm receives during the collusive punishment phase depends on the syndication offers made by the price deviator. (If any firm accepts its syndication offer, play proceeds to the Bertrand reversion phase.) We call this case *insufficient offers*; roughly, offers are insufficient if they do not compensate the other firms for the difference in future profits between the collusive punishment and Bertrand reversion phases.

- If the price deviator's syndication offers are great enough that dynamic rewards are not sufficient to deter firms from accepting offers of syndication, then we simply proceed to the Bertrand reversion phase regardless of which syndication offers are accepted. We call this case *sufficient offers*.[42]

It is immediate that the conjectured equilibrium delivers a price of $p^\star$ in each period. We now explain why the prescribed strategies constitute a subgame-perfect Nash equilibrium.

Responding to Syndication Offers

It is straightforward that the prescribed actions regarding accepting or rejecting syndication offers are best responses after equilibrium play and in the uniformly low and sufficient offers cases. It is only in the case of insufficient offers that (non-price-deviating) firms are required to accept/reject syndication offers in a way that does not maximize in-period profits. In the insufficient offers case, total future industry profits in the collusive punishment phase are the discounted value of the collusive punishment price less the efficient cost of production, and these are distributed in such a way that each firm finds it incentive compatible to reject its offer of syndication.

Responding to Price Offers

It is immediate that each short-lived buyer b_t is acting optimally, as b_t always chooses one of the lowest price offers less than or equal to its reservation price v.

[42] See Appendix A.3 for a formal characterization of when offers are uniformly low, insufficient, and sufficient.

Deviating on Price or Syndication Offers in the Collusive Punishment Phase

We begin by verifying that during a collusive punishment phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most q from the buyer. Thus, firm f's profit in-period is at most $q - c(1, \varphi) \leq c(1, \varphi) - c(1, \varphi) = 0$ as $q = \min\{v, c(1, \varphi)\}$. Moreover, firm f's profits in every future period will be 0, as play will enter a new collusive punishment phase in which firm f obtains 0 profits each period. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most 0. On the other hand, firm f enjoys a non-negative continuation value by not deviating; consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during a collusive punishment phase. Offers are only sufficient if they compensate the other firms for future play switching to the Bertrand reversion phase instead of a collusive punishment phase. Thus, if the net present value of future profits in the collusive punishment phase is large enough—which is guaranteed by our assumption that $\delta \geq \frac{1}{2}$—then the cost of sufficient offers is high enough that a firm would be better off engaging in lone production than making sufficient offers.[43]

Deviating on Price or Syndication Offers in the Cooperation Phase

Finally, we verify that during the cooperation phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most $p^\star$ from the buyer. Thus, firm f's profit in-period is at most $p^\star - c(1, \varphi)$.

[43]For further details on why we require $\delta \geq \frac{1}{2}$, see Appendix A.3.3.

Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most $p^\star - c(1, \varphi)$. On the other hand, firm f enjoys profits each period of $\varphi(p^\star - c(1, 1))$ by not deviating. Consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^\star - c(1, 1)) \geq p^\star - c(1, \varphi),$$

which holds as $p^\star \leq \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$ by construction. The reasoning for why making sufficient offers is non-optimal is the same as in the analysis of the collusive punishment phase above.

Maximality of $p^\star$

Having shown that $p^\star$ is sustainable in equilibrium, all that remains is proving that no higher price can be sustained. It is straightforward that no price higher than v can be sustained, since buyers would reject such price offers. Meanwhile, when $p^\star < v$, the price $p^\star$ is exactly the highest price at which no firm wishes to deviate within-period and engage in lone production. Thus if the buyer were to accept an offer of $p > p^\star$ each period, at least one firm is not playing a best response: Any firm f receiving its fair share (or less) of industry profits at price p could offer a price slightly less than p (which would induce the buyer to choose f) and engage in lone production; by the construction of $p^\star$, this would be profitable for f. Thus we see that $p^\star$ is indeed the highest sustainable price.

3.4 Fixed Firm Capacity vs. Fixed Industry Capacity

Our result that the maximally collusive price may be decreasing in market concentration (and hence, increasing in the number of firms) relies on the assumption that per-firm productive capacity is decreasing in the number of firms; in many syndicated markets, such as the market for IPOs, this assumption seems natural.[44] In other syndicated markets, new entrants may

[44] For instance, Khanna et al. (2008) argued that IPOs require specialized labor, and the supply of that labor within the industry is essentially fixed. Furthermore, IPO shares are primarily purchased by institutional

bring new capacity.

An alternative specification of our model would thus assign each firm a fixed capacity regardless of the number of firms present. We show in Appendix C that with fixed firm capacity, the highest sustainable price becomes decreasing in the number of firms, but nevertheless remains bounded above the marginal cost of efficient production. Moreover, with fixed firm capacity, industry profits may be *increasing* in the number of firms. Just as in our main model, the necessity of syndication for efficient production pushes prices above competitive levels.

Similarly, we could consider a model in which the cost of lone production increases as the number of firms grows, but approaches a finite limit. In that case, the maximally collusive price again remains bounded above the marginal cost of efficient production even as the number of firms grows large.[45]

3.5 Costs of Syndicate Formation

In our model, efficient production within each period requires that all firms participate in the syndicate. In practice, syndicates typically do not include all firms in an industry; for instance, IPO underwriting syndicates typically do not include all active IPO underwriters. A natural extension of our model would be to include a "coordination cost" that is increasing and weakly convex in the number of firms; with these coordination costs, the efficient syndicate would no longer necessarily include all firms. Nevertheless, in this setting, firms would still be able to maintain high prices even as the number of firms grows large; the wedge between the cost of efficient (syndicated) production and the cost of lone production would still allow

investors (Jenkinson et al., 2018), suggesting a relatively fixed pool of capital available to IPO underwriters.

[45] However, while the maximally collusive price is always weakly decreasing in the number of firms with fixed firm capacity, this may not be true in the more general setting where the cost of lone production approaches a finite limit.

firms to punish price-deviators in-period, thus enabling collusion.[46,47]

4 Prices, Profits, and Capacity

We now consider the question of how the highest sustainable price and industry profits in a maximally collusive equilibrium vary as a function of the productive capacity k. In standard industrial organization models, industry profits are increasing in the productive efficiency of firms—in our setting, however, this is not necessarily the case. Indeed, for a large class of cost functions, industry profits in our setting are strictly *decreasing* in the productive capacity k.

Proposition 3. *If $c(s, \varphi) - c(s, 1)$ is convex in s for all $\varphi \in (0, 1 - \delta)$,[48] then the highest sustainable price $p^\star$ and industry profits in a maximally collusive equilibrium are decreasing in productive capacity k.*

Increasing the productive capacity affects the highest sustainable price, $p^\star$, through two channels: First, increasing productive capacity lowers the cost of efficient joint production, making collusion more profitable. But increasing productive capacity also lowers the cost of lone production, making price deviation and lone production more profitable. As the sustainability of collusion depends on the relative profitability of these two options,[49] increasing capacity could potentially make collusion easier or harder to sustain. When the difference between the cost of lone production ($c(s, \varphi)$) and the cost of efficient joint production ($c(s, 1)$) is increasing and convex in the quantity produced (s), the second effect dominates; this makes

[46] Note that depending on how coordination costs are modeled, it is possible that the optimal syndicate could involve fewer and fewer firms as the overall number of firms grows; in the limit, this would make syndicate formation inefficient, causing the wedge between the cost of efficient production and the cost of lone production to vanish. Then, however, the cost of efficient production approaches ∞ as the number of firms grows large and so "no-trade" becomes the unique equilibrium.

[47] If each firm has a fixed capacity, as in Section 3.4, then coordination costs would imply a constant optimal syndicate size; in this case, regardless of the number of firms, there would exist a wedge between the cost of efficient production and the cost of lone production, which would enable collusive strategies of the type described here.

[48] For instance, all cost functions of the form $c(s, m) = s\left(\frac{s}{m}\right)^\alpha$, where $\alpha > 0$, satisfy this condition.

[49] Recall from our derivation of $p^\star$ in Section 3.3 that $p^\star$ is chosen so that price-deviating and then engaging in lone production is unprofitable.

collusion harder to sustain and thus the highest sustainable price falls as productive capacity increases.

Even though the highest sustainable price $p^\star$ falls as productive capacity increases, one might still intuitively expect that increasing the productive capacity k would enhance industry profits in a maximally collusive equilibrium. However, as just described, when the difference between the cost of lone production ($c(s,\varphi)$) and the cost of efficient joint production ($c(s,1)$) is increasing and convex in the quantity produced s, the highest sustainable price $p^\star$ falls as productive capacity increases. Moreover, as productive capacity increases, the highest sustainable price (and thus industry revenues) drops faster than the cost of efficient production. Thus, industry profits decline as productive capacity increases.

5 Contracting over Production Shares

We now consider an extension of our model in which each syndication offer to a non-leading firm g specifies not only the fee that g receives from participating, but also the share of production that g completes. Under this form of contracting, in Step 3 of the extensive form stage game, the syndicate leader ℓ offers each other firm g a contract (s_t^g, w_t^g). If g accepts this syndication offer, it receives a *fee* of w_t^g from ℓ (as before) and completes a *production share* s_t^g. The stage game payoffs (where, as before, the set of firms who accept the offer of syndication is denoted by G_t) are now as follows:

1. The payoff for ℓ is $p_t^\ell - c\Big(1 - \sum_{g\in G_t \smallsetminus \{\ell\}} s_t^g, \varphi k\Big) - \sum_{g\in G_t \smallsetminus \{\ell\}} w_t^g$, i.e., the price paid by the buyer less both the cost of ℓ's production and the fees paid to other firms.

2. The payoff for $g \in G_t \smallsetminus \{\ell\}$ is $w_t^g - c(s_t^g, \varphi k)$, i.e., the fee paid to g less the cost of g's production.

3. The payoff for $h \in F \smallsetminus G_t$ is 0.

Surprisingly, the highest sustainable price is the same as when firms are unable to contract over production shares (Theorem 1).

Theorem 2. *If syndication offers specify both production shares and fees, then for $\delta \geq \frac{1}{2}$, the highest sustainable price is given by $p^\star$, as defined in Theorem 1; consequently, either $p^\star$ is strictly single-troughed (and thus non-monotonic) in φ or $p^\star = v$ everywhere.*

We give a full proof of Theorem 2 in Appendix B.2. To prove that $p^\star$ is sustainable when syndication offers specify production shares, we construct an equilibrium that is very similar to the one we constructed in Section 3. In particular, the equilibrium supporting Theorem 2 has the same set of phases as in Section 3.3 and the circumstances under which play transitions from one phase to another are comparable.

The sustainability of collusion depends on the relative profitability for each firm of colluding versus price-deviating and then engaging in lone production. Recall from our derivation of $p^\star$ in Section 3.3 that $p^\star$ is chosen so that price-deviating and then engaging in lone production is unprofitable. Because price-deviating and then engaging in lone production does not involve multi-firm syndicates, changing the contracting structure between syndicate leaders and non-leading firms does not affect $p^\star$ directly.

Changing the contracting structure does make recruiting syndicate members after a price deviation easier. Thus, we might worry that collusion might not be sustainable because a different type of deviation would become attractive: price-deviating and then building a syndicate. However, so long as $\delta \geq \frac{1}{2}$, it is still more costly for a price deviator to make sufficient offers (and thus recruit a syndicate) than to engage in lone production; see Appendix B.2 for details.

6 Heterogeneous Firms and Market Entry

Our analysis so far has assumed that firms are homogenous, i.e., that each firm has the same productive capacity. As we show in Appendix D, Theorem 2 naturally generalizes to the case in which firms may have different productive capacities; but now, the share of per-period profits allocated to each firm depends on its size. In particular, the maximally collusive equilibrium allocates profits among firms so that each firm (weakly) prefers its equilibrium strategy to undercutting on price and engaging in lone production. Accordingly, larger firms are allocated larger per-period profits than smaller ones, as larger firms have a lower cost of lone production.

Our analysis in Appendix D also allows us to consider the effect of market entry: Surprisingly, entry by a firm with a small amount of productive capacity *raises* (rather than lowers) the highest sustainable price. The key idea is that a small firm is not able to profitably undercut on price and engage in lone production as the highest sustainable price will be below a small firm's cost of lone production. Thus, the only effect of entry by a small firm with additional capacity is to increase the amount of productive capacity available to the syndicate; this makes colluding at any given price more profitable (as efficient production is now less costly), while having no effect on any existing firm's payoff from undercutting on price and engaging in lone production.

7 Conclusion

Our results show that in markets with syndication, classical industrial organization intuitions are not always valid: Decreasing market concentration can *raise* prices by strengthening firms' abilities to punish a deviator in-period by refusing offers of syndication.[50] Moreover, entry can also *raise* prices; a small entrant cannot credibly threaten to disrupt the collusive equilibrium, but does make collusion more profitable (and thus more attractive) to incumbent firms. Thus, our analysis suggests that some standard antitrust remedies—such as breaking up firms or facilitating entry—are of questionable use in thwarting collusion in markets with syndication.

Our analysis also adds to the ongoing scholarly debate on whether the IPO underwriting

[50]Although here we work in a complete information environment, in further work (Hatfield et al., 2019b), we show that our conclusions are largely robust to relaxing our assumption that syndication offers are public.

market is collusive and, if so, how collusion persists despite low market concentration in the industry. Our results may offer insight into other features of the financial industry as well: For example, regulatory barriers routinely restrict participation in certain types of investments to investors that meet net worth or financial sophistication requirements. Reducing regulatory barriers should increase the pool of available investors, which corresponds to increasing productive capacity in our model; our Proposition 3 predicts that this increase in productive capacity would likely reduce the highest sustainable price. Surprisingly, our work also suggests that the industry may oppose reducing regulatory barriers even though higher capacity (i.e., more investors) reduces the total cost of production: the decrease in the highest sustainable price from higher capacity may more than offset the lower costs due to increased capacity. Another solution suggested by our work here would be to eliminate the “bake-off” procedure for allocating an IPO and instead have firms submit bidding schedules for portions of the IPO to underwrite.

Finally, our work also highlights the importance of considering the full extensive form of firm interactions in industrial organization settings. Many industries are characterized by repeated, complex interactions that are best modeled as repeated extensive form games, such as IPO underwriting, debt origination, municipal auctions followed by horizontal subcontracting between bidders, and real estate transactions with agent selection; further exploration of repeated extensive form games is thus crucial to understanding subtle but important strategic interactions in these, and many other, markets.

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A Proof of the Main Theorem

A.1 Bertrand Reversion Nash Equilibrium

In this section, we construct a Bertrand reversion Nash equilibrium which suffices to prove Proposition 1, which we restate here for reference.

Proposition 1. *There exists a subgame-perfect Nash equilibrium of the stage game, i.e., the Bertrand reversion Nash equilibrium, in which each firm obtains a payoff of* 0*, its lowest individually rational payoff.*

In the Bertrand reversion Nash equilibrium, each firm f offers a price $p_t^f = c(1,1)$, which is exactly the cost of producing the good under full participation in the syndicate. The buyer then chooses each firm as syndicate leader with equal probability. The syndicate leader then offers each non-leader firm g a fee $w_t^g = c(\varphi, \varphi)$ equal to g's cost of production (assuming all syndication offers are accepted). Each firm $g \in F \setminus \{f\}$ accepts this offer. Under this behavior, each firm in the syndicate other than f then incurs production costs of $c(\varphi, \varphi)$ and thus breaks even. Moreover, the syndicate leader also breaks even as he obtains $c(1,1) = |F|c(\varphi, \varphi)$ from the buyer, he incurs production costs of $c(\varphi, \varphi)$, and he pays $(|F| - 1)c(\varphi, \varphi)$ in total to the syndicate, leaving him with exactly 0 in profit.[51]

If any firm makes an offer other than $c(1,1)$ to the buyer, the buyer chooses the lowest offer.[52] Firms' responses to syndication offers do not depend on the set of offers made to the buyer. If the syndicate leader offers a fee of $c(\varphi, \varphi)$ to each other firm, then each other firm accepts this offer. If the syndicate leader offers a fee other than $c(\varphi, \varphi)$ to

[51] Recall that $c(\cdot, \cdot)$ is homogeneous of degree one.

[52] If there are multiple lowest offers, the buyer chooses each with equal probability.

any firm, then within-period continuation play can follow any profile of actions for the other firms $g \neq f$ that constitutes a Nash equilibrium of the within-period continuation game.[53] Note, however, that regardless of the equilibrium play after a fee other than $c(\varphi, \varphi)$ has been offered to some firm, the syndicate leader f's profits are no greater than $p^f - c(\varphi, \varphi) - (|F| - 1)c(\varphi, \varphi) \leq c(1, 1) - |F|c(\varphi, \varphi) = 0$. This follows as no offer greater than $c(1, 1)$ will be accepted by the buyer, and no firm will accept a syndication offer of less than $c(\varphi, \varphi)$, which is its minimal cost of production as a member of a syndicate. Thus, the syndicate leader will not wish to deviate from the strategy prescribed above. Given his play, other firms will not wish to deviate from their prescribed strategies either.

A.2 Maintaining Collusion with Grim Trigger Strategies When the Market Is Concentrated

We now prove Proposition 2, restated here, that the monopoly price v is sustainable when firms are patient and the number of firms is sufficiently small. Moreover, under these conditions, collusion can be sustained via "grim trigger" strategies: after a deviation in either step of the stage game, play in all subsequent periods reverts to the Bertrand reversion Nash equilibrium described in Section 3.1.

Proposition 2. *If the discount factor is sufficiently high, i.e., $\delta \geq 1 - \varphi$, then there exists a subgame-perfect Nash equilibrium in which each firm offers the monopoly price in every period, i.e., $p_t^f = v$ for any $v \geq c(1, 1)$, for all $f \in F$ and for all t.*

We construct a subgame-perfect Nash equilibrium where every firm offers the monopoly price as follows:

- There are two phases of equilibrium play:

    1. In the *cooperation phase*:

[53] Note that there may be multiple such Nash equilibria, as whether a syndication offer is profitable for a firm may depend on whether other firms accept their syndication offers.

  - Every firm submits the same bid $p = v$,
  - The buyer accepts the lowest price offer so long as one such offer is less than or equal to v. If there are multiple such offers, the buyer accepts each such offer with equal probability. If there are no such offers, the buyer rejects all the offers.
  - Every firm, if it becomes the syndicate leader, offers every other firm $c(\varphi, \varphi)$ to join the syndicate, and
  - Every other firm accepts this offer.

2. In the *Bertrand reversion phase*, firms play the Bertrand reversion Nash equilibrium.

- Under equilibrium play, play continues in the same phase. If, in the cooperation phase, any firm f deviates in the first step or deviates with respect to the prescribed set of offers, then play proceeds to the Bertrand reversion phase. Moreover, if any firm accepts or rejects a syndication offer contrary to the prescribed play, play proceeds to the Bertrand reversion phase.

It is immediate that along the prescribed path of play every firm offers v for all t.

It is also immediate that play in the Bertrand reversion phase is subgame-perfect, as play is a subgame-perfect Nash equilibrium of the stage game (Proposition 1).

In the cooperation phase, an argument analogous to that used to prove Proposition 1 shows that offering $c(\varphi, \varphi)$ to each other firm minimizes the syndicate leader's production costs; moreover, only by offering $c(\varphi, \varphi)$ to each other firm can the syndicate leader possibly obtain positive profits in the future. Thus, offering $c(\varphi, \varphi)$ to each other firm is the optimal action by the syndicate leader during the cooperation phase.

It is immediate that the buyer is acting optimally given the price offers.

Finally, we consider whether any firm will wish to be a price deviator. The expected profits from the equilibrium strategy are given by

$$\frac{1}{1-\delta}\varphi(v - c(1,1)).$$

Again using an argument analogous to that used to prove Proposition 1, we have that offering $c(\varphi, \varphi)$ to each other firm minimizes the syndicate leader's production costs; thus, a price deviator's production costs are given by $c(1, 1)$. Moreover, as we revert to Bertrand competition after a price deviation, profits in all future periods will be 0. Thus, the profits from deviating on price are bounded by

$$v - c(1,1).$$

Thus, so long as $\delta \geq 1 - \varphi$, the strategies described here constitute a subgame-perfect Nash equilibrium.

A.3 Maintaining Collusion with Syndicate Punishments

In this section, we give a formal construction of the strategy profile which sustains the price $p^\star$ defined in Theorem 1, and show that this strategy profile constitutes a subgame-perfect Nash equilibrium. Finally, we show that no subgame-perfect Nash equilibrium can sustain a price higher than $p^\star$.

The equilibrium is constructed as follows:

- There are three phases of equilibrium play:

    1. In the *cooperation phase*,

        - every firm submits the same bid $p = p^\star$,
        - the short-lived buyer accepts one such offer of $p^\star$, choosing each offer with equal probability,

- every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi)$ to every non-leading firm $g \in F \setminus \{\ell\}$ to join the syndicate, and
- every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

2. In the *collusive punishment phase with continuation values* ψ,
   - every firm submits the same bid $q = \min\{c(1, \varphi), v\}$, *the collusive punishment price*,
   - the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,
   - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi) + \psi^g$ to every non-leading firm $g \in F \setminus \{\ell\}$ to join the syndicate, and
   - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
3. In the *Bertrand reversion phase*, firms play the Bertrand reversion Nash equilibrium.

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers. If so, future play depends on the syndication offers that are made.

  Given the fees $\{w^h\}_{h \in F \setminus \{f\}}$ offered by f, we say that a set $H \subseteq F \setminus \{f\}$ is *internally consistent* if, for all $h \in H$, we have that $w^h - c\Big(\frac{1}{|H|+1}, \varphi\Big) \geq 0$; that is, a set H is internally consistent if every firm in H weakly prefers to accept its syndication offer (ignoring payoffs in future periods), assuming that only the firms comprising H accept their syndication offers. Note that, trivially, the empty set is internally consistent. Furthermore, there is a largest internally consistent set in the superset sense. This follows from the fact that, if both H and $\hat{H}$ are internally consistent, then $H \cup \hat{H}$ is internally consistent. To see this, note that for all $h \in H$, we have that $w^h - c\Big(\frac{1}{|H|+1}, \varphi\Big) \geq 0$, implying that $w^h - c\Bigg(\frac{1}{|H \cup \hat{H}|+1}, \varphi\Bigg) \geq 0$, as the cost function is decreasing in the production share; similarly, for all $\hat{h} \in \hat{H}$, we have that $w^{\hat{h}} - c\Bigg(\frac{1}{|H \cup \hat{H}|+1}, \varphi\Bigg) \geq 0$, and so $H \cup \hat{H}$ is internally consistent by definition. Let $\tilde{H}$ denote the largest internally consistent set given the fees $\{w^h\}_{h \in F \setminus \{f\}}$.

In equilibrium, future play will depend on the surplus that can be captured by the largest internally consistent set $\tilde{H}$, which is given by $\sum_{h \in \tilde{H}} \Big(w^h - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)\Big)$. Based on this sum, we categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

Uniformly Low Offers: $\sum_{h \in \tilde{H}} \Big(w^h - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)\Big) = 0$**.** In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production (given that other firms are rejecting their syndication offers).[54] Thus, every non-leading firm rejects its offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{h \in \tilde{H}} \Big(w^h - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)\Big) \leq \frac{\delta}{1-\delta}(q - c(1,1))$**.** In this case, absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \dfrac{w^h - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)}{\sum_{g \in \tilde{H}} \Big(w^g - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)\Big)} (q - c(1,1)) & h \in \tilde{H} \\ 0 & h \in F \setminus \tilde{H}. \end{cases}$$

Sufficient Offers: $\sum_{h \in \tilde{H}} \Big(w^h - c\Big(\frac{1}{|\tilde{H}|+1}, \varphi\Big)\Big) > \frac{\delta}{1-\delta}(q - c(1,1))$**.** In this case, play will enter the Bertrand reversion phase in the next period regardless of each non-leading

[54] In fact, rejecting its syndication offer is a best response for each non-leading firm even if every (other) firm in $\tilde{H}$ accepts its offer of syndication.

firm's behavior. In period, each non-leading firm h accepts its syndication offer if and only if the firm is in $\tilde{H}$. Thus, each firm accepts its syndication offer if and only if that offer is (weakly) profitable within-period, given the actions of other firms. This is optimal, as future profits will be 0 for every firm, regardless of its actions, as play enters the Bertrand reversion phase.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

Figure 2 provides an automaton representation of the subgame-perfect Nash equilibrium described here.

It is immediate that the conjectured equilibrium delivers a price of $p^\star$ in each period. We now verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium.

A.3.1 Responding to Syndication Offers

We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each non-leading firm to accept its syndication offer.[55] It is also immediate that, in the case of uniformly low offers, it is a best response for each non-leading firm to reject its syndication offer.[56] Finally, it is immediate that, in the case of sufficient offers, each non-leading firm plays a best response; each non-leading firm only accepts its syndication offer if accepting provides a non-negative payoff in this period, and play continues to the Bertrand reversion phase regardless of the firm's actions.

[55]This follows as each syndication offer provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, in which the firm's future payoffs are 0.

[56]To see this, consider two cases: If $\tilde{H} = \varnothing$, then $w^h - c\left(\frac{1}{2}, \varphi\right) < 0$ for all $h \in H$ (as otherwise $\{h\}$ would be internally consistent); thus, given that no other firm is accepting, every firm strictly prefers rejecting. If $\tilde{H} \neq \varnothing$, since $\sum_{h \in \tilde{H}} \left(w^h - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right)\right) = 0$, we must have that $w^h - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right) \leq 0$ for all $h \in H$; thus, given that no other firm is accepting, every firm weakly prefers rejecting.

We now show that, in the case of insufficient offers, it is a best response for each non-leading firm to reject its offer of syndication:

- First, consider a firm $h \in \tilde{H}$. We first calculate the total payoff for h from accepting its offer. This is given by

$$w^h - c\left(\frac{1}{2}, \varphi\right) \leq w^h - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right),$$

  because play reverts to the Bertrand reversion phase if h accepts its offer, in which case h will earn 0 future profits.[57] Meanwhile, the total payoff for h in the continuation game from rejecting the offer is

$$\begin{aligned}\frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta}\left(\frac{w^h - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right)}{\sum_{g\in\tilde{H}}\left(w^g - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right)\right)}(q - c(1,1))\right) \\ &\geq w^h - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right),\end{aligned}$$

  where the inequality follows from the fact that $\sum_{g\in\tilde{H}}\left(w^g - c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right)\right) \leq \frac{\delta}{1-\delta}(q - c(1,1))$, as we are in the insufficient offers case.[58]

- Second, we consider a firm $h \in F \setminus (\tilde{H} \cup \{f\})$. The total payoff for h from accepting its offer is given by

$$w^h - c\left(\frac{1}{2}, \varphi\right) \leq w^h - c\left(\frac{1}{|\tilde{H} \cup \{h\}|+1}, \varphi\right) < 0,$$

  where the second inequality follows from the fact that $\tilde{H}$ is the largest internally consistent set.[59] Meanwhile, the total payoff for h in the continuation game from

[57] Note that, since the equilibrium calls for each firm to reject its offer of syndication, h expects that, if it accepts its offer of syndication, it will be the only firm to join the syndicate and thus will have production costs of $c\left(\frac{1}{2}, \varphi\right)$.

[58] Note that, since the equilibrium calls for each firm to reject its offer of syndication, h expects that, if it rejects its offer of syndication, play will shift to a collusive punishment phase.

[59] Note that, since the equilibrium calls for each firm to reject its offer of syndication, h expects that, if it accepts its offer of syndication, it will be the only firm to join the syndicate and thus will have production

rejecting the offer is 0, regardless of the actions of other non-leading firms.

Thus, it is a best response for every non-leading firm to reject its syndication offer in the insufficient offers case.

A.3.2 Responding to Price Offers

It is immediate that each short-lived buyer b_t is acting optimally as b_t always chooses one of the lowest price offers less than or equal to its reservation price v.

A.3.3 Deviating on Price or Syndication Offers in the Collusive Punishment Phase

We begin by verifying that, during a collusive punishment phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most q from the buyer. Thus, firm f's profit in-period is at most $q - c(1, \varphi) \leq c(1, \varphi) - c(1, \varphi) = 0$ as $q = \min\{v, c(1, \varphi)\}$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most 0. On the other hand, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating; consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during a collusive punishment phase. Note that sufficient offers require that the price deviator provide the firms in $\tilde{H}$ with *dynamic compensation* totaling at least $\frac{\delta}{1-\delta}(q - c(1, 1))$ above their costs of production (assuming that, as the equilibrium specifies, all the firms in $\tilde{H}$ accept and all the firms not in $\tilde{H}$ reject). Thus, the in-period

costs of $c(\frac{1}{2}, \varphi)$.

payoff to the deviating firm f is at most

$$\begin{aligned}\underbrace{q}_{\text{Price}} - \underbrace{(|\tilde{H}|+1)c\left(\frac{1}{|\tilde{H}|+1}, \varphi\right)}_{\substack{\text{Total cost of production}\\ \text{when firms in } \tilde{H} \text{ participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,1))}_{\substack{\text{Dynamic compensation}\\ \text{required by firms in } \tilde{H}}} &\leq q - \underbrace{c(1,1)}_{\substack{\text{Total cost of production}\\ \text{when all firms participate}}} - \frac{\delta}{1-\delta}(q - c(1,1)) \\ &= \left(1 - \frac{\delta}{1-\delta}\right)(q - c(1,1)) \\ &\leq 0,\end{aligned}$$

where the last inequality follows as $\delta \geq \frac{1}{2}$.[60] In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to 0. By contrast, if firm f continues with equilibrium play, it receives a non-negative payoff. Thus, not deviating is a best response for firm f.

A.3.4 Deviating on Price or Syndication Offers in the Cooperation Phase

Finally, we verify that, during the cooperation phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most $p^\star$ from the buyer. Thus, firm f's profit in-period is at most $p^\star - c(1, \varphi)$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most $p^\star - c(1, \varphi)$. On the other hand, firm f enjoys profits each period of $\varphi(p^\star - c(1, 1))$ by not deviating. Consequently, it is not

[60] Our result also obtains for some discount factors less than $\frac{1}{2}$, but assuming that $\delta \geq \frac{1}{2}$ greatly simplifies our presentation here. In particular, assuming that $\delta \geq \frac{1}{2}$ is sufficient to guarantee that the future payoffs in the collusive punishment phase available to firms other than f are sufficiently large that f will prefer to engage in lone production rather than pay sufficient fees to entice firms to join f's syndicate. When δ falls slightly below $\frac{1}{2}$, we can obtain the same highest sustainable price, but this requires carefully choosing firms' conjectures about how other firms will respond to off-path syndication offers after a deviation.

profitable for f to deviate and make uniformly low or insufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^\star - c(1,1)) \geq p^\star - c(1,\varphi),$$

which holds as $p^\star \leq \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$ by construction.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during the cooperation phase. Recall that sufficient offers require that the price deviator provide the firms in $\tilde{H}$ with dynamic compensation totaling at least $\frac{\delta}{1-\delta}(q - c(1,1))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{p^\star}_{\text{Price}} - \underbrace{c(1,1)}_{\substack{\text{Total cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,1))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}}. \tag{1}$$

In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to that given by (1). By contrast, if firm f continues with equilibrium play, firm f enjoys profits each period of $\varphi(p^\star - c(1,1))$. Consequently, it is not profitable for f to deviate and make sufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^\star - c(1,1)) \geq p^\star - c(1,1) - \frac{\delta}{1-\delta}(q - c(1,1)),$$

which reduces to

$$p^\star \leq \frac{(1-\delta)c(1,1) + \delta(q - c(1,1)) - \varphi c(1,1)}{1-\delta-\varphi}.$$

There are now two cases to consider, depending on $q = \min\{c(1,\varphi), v\}$: In the first case, $q = c(1,\varphi)$. Thus, as $p^\star = \min\{\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}, v\} \leq \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$, it is not profitable for

f to deviate by making sufficient offers so long as

$$\begin{aligned}
\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi} &\leq \frac{(1-\delta)c(1,1)+\delta(c(1,\varphi)-c(1,1))-\varphi c(1,1)}{1-\delta-\varphi} \\
(1-\delta)c(1,\varphi) &\leq (1-\delta)c(1,1)+\delta(c(1,\varphi)-c(1,1)) \\
(2\delta-1)c(1,1) &\leq (2\delta-1)c(1,\varphi),
\end{aligned}$$

which holds since $\delta \geq \frac{1}{2}$ and $c(1,1) < c(1,\varphi)$.

In the second case, $q = v$, which implies that $p^\star = v$.[61] Thus, it is not profitable for f to deviate by making sufficient offers so long as

$$\begin{aligned}
v &\leq \frac{(1-\delta)c(1,1)+\delta(v-c(1,1))-\varphi c(1,1)}{1-\delta-\varphi} \\
(1-\delta-\varphi)v &\leq \delta v+(1-2\delta-\varphi)c(1,1) \\
(2\delta+\varphi-1)c(1,1) &\leq (2\delta+\varphi-1)v.
\end{aligned}$$

This holds since $\delta \geq \frac{1}{2}$, $\varphi > 0$, and $v \geq c(1,1)$.

Thus, for $\delta \geq \frac{1}{2}$, $p^\star$ can be sustained.

A.3.5 Maximality of $p^\star$

It now remains to show that no price higher than $p^\star$ can be sustained. There are two cases to consider, depending on whether $p^\star = v$ or $p^\star = \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$: In the former case, no price greater than $p^\star = v$ can be sustained as no buyer will accept an offer higher than v.

In the latter case, suppose there existed an equilibrium in which the buyer accepted an offer of $p > p^\star$ each period. We show that at least one firm is not playing a best response: The total industry profits generated each period are at most $p - c(1,1)$, and so the total expected industry profits are at most $\frac{1}{1-\delta}(p - c(1,1))$. Thus, there must exist at least one firm f with

[61] Note that when $1-\delta-\varphi > 0$, we may calculate that $p^\star \geq q$, as $\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi} - c(1,\varphi) = \frac{\varphi(c(1,\varphi)-c(1,1))}{1-\delta-\varphi} > 0$, and so $\min\{\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}, v\} - \min\{c(1,\varphi), v\} \geq 0$.

total expected profits of at most $\frac{1}{1-\delta}\varphi(p - c(1, 1))$. If firm f deviated by offering a price of $p - \epsilon$ and engaging in lone production, f's in-period profits approach $p - c(1, \varphi)$ as $\epsilon \to 0$. No matter the behavior of other firms in subsequent play, f can guarantee itself non-negative profits in each subsequent period.[62] Therefore, firm f has profits of deviating of at least $p - c(1, \varphi) > \frac{1}{1-\delta}\varphi(p - c(1, 1))$, its profits from not deviating, as $p > p^\star = \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$.

A.3.6 Behavior of $p^\star$

We now show that $p^\star$ is quasiconvex in order to show that $p^\star$ is either single-troughed or flat. In the region where $p^\star$ is less than v, we have that the second derivative of $p^\star$ with respect to φ is given by

$$\frac{\partial^2 p^\star}{\partial \varphi^2} = \frac{(1-\delta)\frac{\partial^2 c(1,\varphi)}{\partial \varphi^2}}{1-\delta-\varphi} + \frac{2}{1-\delta-\varphi}\underbrace{\frac{(1-\delta)\Big(c(1,\varphi) - c(1,1) + (1-\delta-\varphi)\frac{\partial c(1,\varphi)}{\partial \varphi}\Big)}{(1-\delta-\varphi)^2}}_{\frac{\partial p^\star}{\partial \varphi}},$$

which is positive at any critical point of $p^\star$: The first term is positive as the cost function is convex in its second argument (and $\delta < 1 - \varphi$) and the second term must be 0 at any critical point. Thus, $p^\star$ is quasiconvex over the region where $p^\star < v$. It is then immediate that $p^\star$ is quasiconvex over its entire domain as it is the minimum of a quasiconvex function and a constant.

Finally, for $\varphi \in (0, 1-\delta)$, we have that

$$p^\star = \min\left\{\frac{(1-\delta)c(1,\varphi) - \varphi c(1,1)}{1-\delta-\varphi}, v\right\}$$

and it is immediate that

$$\lim_{\varphi \to 0} \frac{(1-\delta)c(1,\varphi) - \varphi c(1,1)}{1-\delta-\varphi} = \infty$$

[62] For example, f could offer a price of $c(1, \varphi)$ and, if chosen by the buyer, offer a syndication fee of 0 to all other firms and, if not chosen, reject all syndication offers.

as $\lim_{\varphi \to 0} c(1, \varphi) = \infty$ by assumption. Further,

$$\lim_{\varphi \to 1-\delta} \frac{(1-\delta)c(1,\varphi) - \varphi c(1,1)}{1-\delta-\varphi} = \infty.$$

Thus, if v is sufficiently high, i.e., v exceeds the minimum of $\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$ for $\varphi \in (0, 1-\delta)$, the highest sustainable price $p^\star$ is single-troughed in φ; otherwise $p^\star = v$.

B Proofs of Other Results

B.1 Proof of Proposition 3

We first show that industry profits in the maximally collusive equilibrium are decreasing in k. It is easy to verify that price is now given by:

$$p^\star = \frac{(1-\delta)c(1,\varphi k) - \varphi c(1,k)}{1-\delta-\varphi}.$$

Industry profits per period are thus

$$\Pi \equiv \frac{(1-\delta)c(1,\varphi k) - \varphi c(1,k)}{1-\delta-\varphi} - c(1,k) = \frac{1-\delta}{1-\delta-\varphi} k \left(c\left(\frac{1}{k}, \varphi\right) - c\left(\frac{1}{k}, 1\right) \right).$$

where the equality follows from the fact that the cost function is homogeneous of degree 1. Differentiating profits with respect to k, and then multiplying by $\frac{1-\delta-\varphi}{1-\delta}$ gives

$$\frac{1-\delta-\varphi}{1-\delta} \frac{\partial \Pi}{\partial k} = \left(c\left(\frac{1}{k}, \varphi\right) - c\left(\frac{1}{k}, 1\right) \right) - \frac{1}{k} \left(c_s\left(\frac{1}{k}, \varphi\right) - c_s\left(\frac{1}{k}, 1\right) \right)$$

Letting $g(x) = c(x, \varphi) - c(x, 1)$ and $x = \frac{1}{k}$, we have that

$$
\begin{aligned}
\frac{1-\delta-\varphi}{1-\delta}\frac{\partial \Pi}{\partial k} &= g(x) - xg'(x) \\
&= g(x) - g(0) - (x-0)g'(x) \\
&< 0,
\end{aligned}
$$

where the second equality follows from the from the fact that $c(0, y) = 0$ for all $y \geq 0$, and the inequality follows from the convexity assumption of the theorem.

Since both the cost of efficient joint production and industry profits in the maximally collusive equilibrium are decreasing in k, the highest sustainable price must also be decreasing in k.

B.2 Proof of Theorem 2

To show that $p^\star$ is the highest sustainable price, we construct an equilibrium of the following form:[63]

- There are three phases of equilibrium play:
    1. In the *cooperation phase*,
        - every firm submits the same bid $p = p^\star$,
        - the short-lived buyer accepts one such offer of $p^\star$, choosing each offer with equal probability,
        - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi k)$ to every non-leading firm $g \in F \setminus \{\ell\}$ for agreeing to perform φ of production, and
        - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
    2. In the *collusive punishment phase with continuation values ψ*,

[63] It is immediate that, when $\varphi \in (1-\delta, 1]$, we can sustain collusion exactly as in the proof of Theorem 1.

– every firm submits the same bid $q = \min\{c(1, \varphi), v\}$,

– the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,

– every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi) + \psi^g$ to every non-leading firm $g \in F \setminus \{\ell\}$ for agreeing to perform φ of production, and

– every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

3. In the *Bertrand reversion phase*, firms play the Bertrand reversion Nash equilibrium.[64]

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers. If so, future play depends on the sum over the non-leading firms of the (positive) difference between the syndication fee w^g offered to each firm g and the cost to that firm of doing s^g of the project, $\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \varphi))^+$.[65] Based on this sum, we categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

Uniformly Low Offers: $\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \varphi))^+ = 0$. In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production. Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \varphi))^+ \leq \frac{\delta}{1-\delta}(c(1, \varphi) - c(1, 1))$. In this case,

[64] Here, in the Bertrand reversion Nash equilibrium, the syndicate leader offers every other firm $c(\varphi, \varphi)$ for agreeing to perform φ of the production.

[65] Here, $(x)^+ \equiv \max\{0, x\}$.

absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{\left(w^h - c\left(s^h, \varphi\right)\right)^+}{\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \varphi))^+} (q - c(1,1)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \varphi))^+ > \frac{\delta}{1-\delta}(q - c(1,k))$. In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq c(s^g, \varphi)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

The proof that this strategy profile is a subgame-perfect Nash equilibrium and that it attains the highest sustainable price of any subgame-perfect Nash equilibrium then follows as in the proof of Theorem 1 given in Appendix A.[66]

C Collusion with Fixed Firm Capacity

Our baseline model assumes that the total capacity of the industry is fixed as the number of firms grows; thus, the capacity per firm falls as the number of firms grows. Here, we consider the alternative case in which firm capacity is fixed, and so the total capacity grows as the number of firms grows; we denote by κ the (fixed) capacity of each firm. The equilibrium for the case in which there are $|F|$ firms and each firm has a capacity κ corresponds exactly

[66] Note that in this case one could construct equilibria that, after a price deviation, reward firms for rejecting syndication offers by lowering the amount of production required of those firms in future periods. However, it is more efficient to reward firms exclusively through fees; any vector of continuation payoffs that can be achieved by a combination of particular combination of quantity offers and fees can be weakly improved upon for firms being rewarded in the collusive punishment phase by having each firm perform an equal share and modifying fees appropriately.

to the case in which there are $|F|$ firms and total capacity is $k = |F|\kappa$. Thus, we can use Theorem 1 to derive the following result.

Theorem C.1. *With fixed firm capacity κ, for $\delta \geq \frac{1}{2}$, the highest sustainable price, $p^\star$, is given by*

$$p^\star = \begin{cases} v & \varphi \in [1-\delta, 1] \\ \min\left\{\frac{(1-\delta)c(1,\kappa)-c(\varphi,\kappa)}{1-\delta-\varphi}, v\right\} & \varphi \in (0, 1-\delta). \end{cases}$$

Moreover, $\lim_{\varphi \to 0} p^\star = c(1,\kappa)$.

Theorem C.1 demonstrates that, even in the limiting case in which each firm has a particular fixed capacity κ (regardless of the number of firms), the highest sustainable price is appreciably higher than the cost of production. Just as with Theorem 1, the syndicated nature of the market drives Theorem C.1: if a firm undercuts on price, other firms still have the ability to punish the price deviator in-period by refusing to work with it. Since each firm has a fixed capacity, the cost of lone production no longer increases as the number of firms grows; thus, when there are a large number of firms, the highest sustainable price is exactly the cost of lone production.

When each firm has a fixed capacity κ, the highest sustainable price is now weakly decreasing in the number of firms.[67] Note, however, that the cost of efficient production is

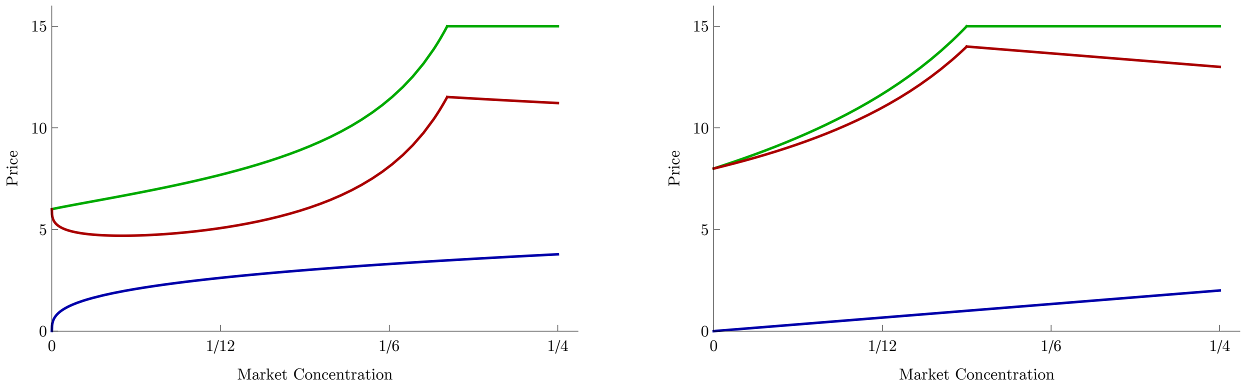
[67] When $p^\star < v$, we calculate that

$$\frac{\partial p^\star}{\partial \varphi} = \frac{(1-\delta)c(1,\kappa) - c(\varphi,\kappa) - (1-\delta-\varphi)c_s(\varphi,\kappa)}{(1-\delta-\varphi)^2}.$$

Note that $c(1-\delta,\kappa) - c(\varphi,\kappa) \geq (1-\delta-\varphi)c_s(\varphi,\kappa)$ as $c(s,m)$ is convex in its first argument. Thus

$$\begin{aligned} \frac{\partial p^\star}{\partial \varphi} &\geq \frac{(1-\delta)c(1,\kappa) - c(\varphi,\kappa) - (c(1-\delta,\kappa) - c(\varphi,\kappa))}{(1-\delta-\varphi)^2} \\ &= \frac{(1-\delta)c(1,\kappa) - c(1-\delta,\kappa)}{(1-\delta-\varphi)^2} \\ &= \frac{c(1-\delta,(1-\delta)\kappa) - c(1-\delta,\kappa)}{(1-\delta-\varphi)^2} \\ &> 0 \end{aligned}$$

where the last equality follows from the fact that $c(s,m)$ is homogeneous of degree 1 and the last inequality follows from the fact that $c(s,m)$ is decreasing in its second argument.



(a) $\alpha = \frac{1}{3}$.

(b) $\alpha = 1$.

Figure 3: The highest sustainable price $p^{\star}$ in green, the cost of efficient production in blue, and the markup in red, all as a function of market concentration φ. Here, $c(s, m) = \frac{s}{\alpha}\left(\frac{s}{m}\right)^{\alpha}$, $\kappa = \frac{1}{8}$, $\delta = \frac{3}{4}$, and the maximum price that the buyer is willing to pay is $v = 15$. In Figure 3a, the markup is non-monotonic in the number of firms, while in Figure 3b, the markup is monotonically decreasing in the number of firms once the industry is sufficiently unconcentrated.

also now decreasing in the number of firms. Thus, the natural metric to evaluate the effect of the number of firms on competition in this setting is industry profits as the number of firms changes; we can calculate these profits as the markup over the cost of efficient production, i.e., $p^{\star} - c(1, k)$ where $k = \frac{\kappa}{\varphi}$. As demonstrated in Figure 3, the effect of the number of firms on industry profits is ambiguous and depends on the cost function; however, industry profits never go to 0.

D Heterogeneous Firms

We now extend the model of Section 5 to consider the case in which firms' productive capacities differ.[68] Thus, for each $f \in F$, let κ^f be the *productive capacity controlled by firm* f. It will be helpful to define $\kappa^{\max}$ as the largest share of productive capacity controlled by a single firm, i.e., $\kappa^{\max} \equiv \max_{f \in F}\{\kappa^f\}$. Moreover, the total productive capacity is given by $k = \sum_{f \in F} \kappa^f$.

[68] Here, modeling syndication contracts as specifying both a fee and a production share is natural, since efficient production requires firms with different productive capacities to perform differing production shares.

D.1 Equilibrium Characterization

We now characterize the highest sustainable price as a function of the firms' productive capacities, which we denote $\hat{p}^{\star}(\kappa;\delta)$. To prove that $\hat{p}^{\star}(\kappa;\delta)$ is sustainable, we construct an equilibrium that sustains $\hat{p}^{\star}(\kappa;\delta)$; this equilibrium is very similar to the one constructed in Section 5.

In our constructed equilibrium, if a firm is small enough, it is allocated no surplus in the cooperation phase. This is because, if a firm is small enough, the firm's cost of production will be greater than the highest sustainable price. Accordingly, it will not be profitable for that firm to price-deviate and then engage in lone production. Therefore, no surplus is needed to disincentivize this firm from price-deviating and then engaging in lone production. This frees up additional surplus that can be allocated to larger firms that *will* be tempted to price-deviate and then engage in lone production, enabling the industry to sustain a higher collusive price. We call firms that obtain positive surplus in an equilibrium supporting the highest sustainable price $\hat{p}^{\star}(\kappa;\delta)$ *collusion beneficiaries* and denote the set of collusion beneficiaries as $\hat{F}$.

To prevent a collusion beneficiary f from undercutting on price and engaging in lone production, f's profits from colluding must be large enough that f prefers to adhere to the equilibrium. Consider an equilibrium that sustains the price p and let r^f denote the fraction of surplus allocated to f. In an equilibrium, f must not be tempted to engage in lone production, so the following constraint must hold:

$$\frac{1}{1-\delta} r^f (p - c(1,k)) \geq p - c\Big(1, \kappa^f\Big). \tag{2}$$

Maximizing price subject to constraint (2) for each collusion beneficiary, along with the constraints that $r^f \geq 0$ for all firms and that $\sum_{f \in F} r^f = 1$, yields the highest sustainable price $\hat{p}^{\star}(\kappa;\delta)$, as expressed in Theorem D.1.

Theorem D.1. *Suppose syndication offers specify both production shares and fees and firms* *may have heterogeneous production capacities and $\varphi < 1 - \delta$.*[69] *If*

$$\sum_{f \in F} \max\Big\{v - c\Big(1, \kappa^f\Big), 0\Big\} \leq \frac{1}{1-\delta}(v - c(1,k)), \tag{3}$$

then $\hat{p}^\star = v$ so long as

$$\delta \geq \hat{\delta} \equiv \frac{v - c(1,k)}{v - c(1,k) + \min\{v, c(1, \kappa^{\max})\} - c(1,k)} \in [\frac{1}{2}, 1);$$

otherwise, the highest sustainable price is given by the (unique) solution to

$$\begin{aligned}
\hat{p}^\star &= \frac{(1-\delta)\hat{\varphi}\sum_{f \in \hat{F}} c\Big(1, \kappa^f\Big) - \hat{\varphi}c(1,k)}{1 - \delta - \hat{\varphi}} \\
\hat{F} &= \Big\{f \in F : \hat{p}^\star \geq c\Big(1, \kappa^f\Big)\Big\} \\
\hat{\varphi} &= \frac{1}{|\hat{F}|}
\end{aligned}$$

so long as $\delta \geq \hat{\delta} \equiv \frac{\hat{p}^\star - c(1,k)}{\hat{p}^\star - c(1,k) + c(1,\kappa^{\max}) - c(1,k)} \in [\frac{1}{2}, 1)$.[70]

When Condition (3) is satisfied, firms are able to sustain the monopoly price; Condition (3) ensures that (under efficient joint production) there are sufficient profits at a price of v that profits can be divided among firms so that each firm prefers colluding in each period to undercutting on price and engaging in lone production. This first case represents two scenarios: In the first scenario, even the largest firm is relatively small, i.e., $v < c(1, \kappa^{\max})$. Thus, sustaining v is relatively easy, as no firm has an incentive to undercut on price and engage in lone production, regardless of its profit share. In this scenario, the discount factor threshold $\hat{\delta}$ is simply $\frac{1}{2}$. This scenario holds when the largest firm is not too large and v is sufficiently small.

In the second scenario, at least one firm f is large enough so that $v > c\Big(1, \kappa^f\Big)$. Thus, to sustain v, it is necessary that any firm f such that $v > c\Big(1, \kappa^f\Big)$ obtains positive profits

[69]When the market concentration φ is high enough, i.e., $\varphi \geq 1 - \delta$, monopoly prices can be maintained by traditional Bertrand reversion, "grim trigger," strategies.

[70]See Appendix D.3 for the proof of Theorem D.1.

sufficient to ensure that f prefers its flow of per-period profits to undercutting on price and engaging in lone production today. Condition (3) ensures that there are sufficient profits to distribute among firms so that each firm's per-period profits are large enough to sustain v, i.e., there exists $\rho \in \Delta^F \equiv \{\xi \in \mathbb{R}^F_{\geq 0} : \sum_{f \in F} \xi_f = 1\}$ such that

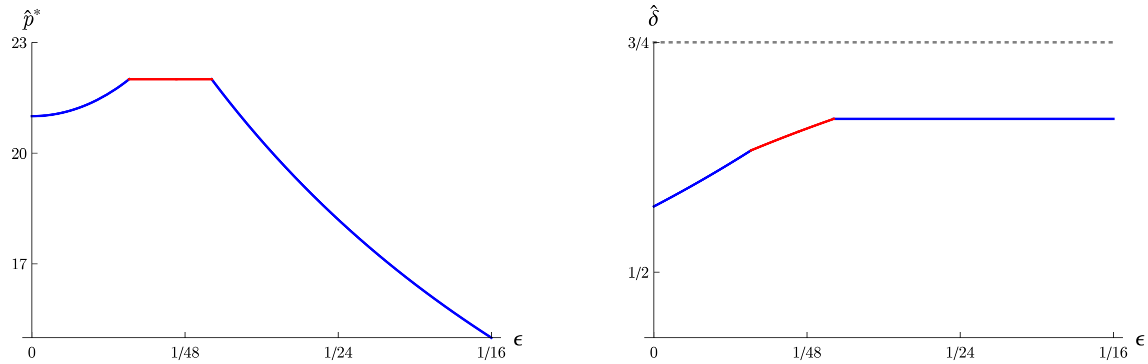
$$\frac{1}{1-\delta}\rho^f(v - c(1,k)) \geq v - c\Big(1, \kappa^f\Big)$$

for every $f \in F$. In this scenario, the discount factor threshold $\hat{\delta}$ is $\frac{v-c(1,k)}{v-c(1,k)+c(1,\kappa^{\max})-c(1,k)}$.

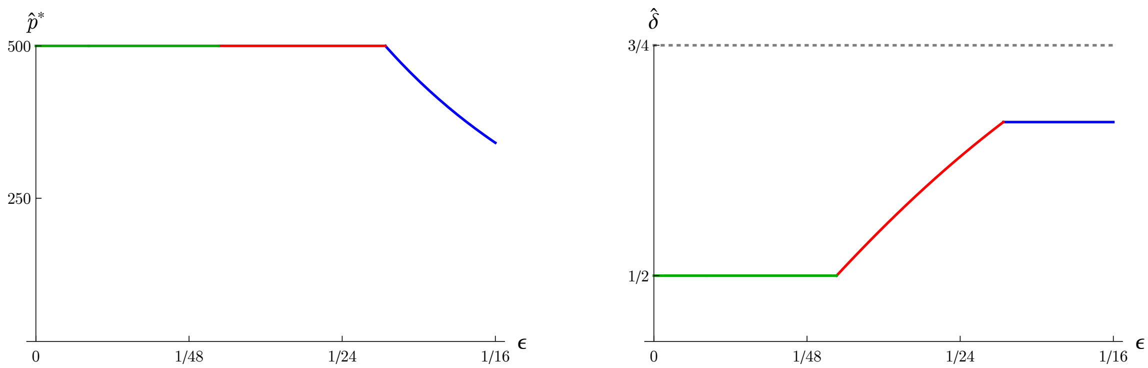
In the second case (i.e., when Condition (3) is not satisfied), there are not sufficient profits to distribute among the lower-cost firms to sustain v. Thus, the highest sustainable price is below v and each firm is allocated per-period profits just sufficient to incentivize that firm to not undercut on price and engage in lone production. Here, $\hat{F}(\kappa, \delta)$ is the set of firms that, at the price $\hat{p}^\star(\kappa; \delta)$, would obtain positive profits by undercutting on price and engaging in lone production; any firm $f \in F \smallsetminus \hat{F}(\kappa, \delta)$ obtains 0 profits in any equilibrium that sustains $\hat{p}^\star(\kappa; \delta)$.

To provide intuition for Theorem D.1, consider the example illustrated in Figure 4; this figure represents an economy with 16 firms, 8 of which are "large" and have productive capacity $\frac{1}{16} + \epsilon$ and 8 of which are "small" and have productive capacity $\frac{1}{16} - \epsilon$. All three scenarios of Theorem D.1 are shown in Figure 4. The left panel of Figure 4a shows the highest sustainable price as a function of ϵ: When the line is blue and firms are nearly homogenous, there are not sufficient profits to distribute among the firms to sustain v; yet, to sustain collusion, each firm must receive a positive share of the profits. As heterogeneity increases, the cost for a small firm to engage in lone production is increasing and the cost for a large firm to engage in lone production is decreasing; but since the cost function is convex in productive capacity, the average cost of lone production, $\varphi \sum_{f \in F} c\Big(1, \kappa^f\Big)$, is increasing, enabling the industry to sustain a higher collusive price.

When the line is red in Figure 4a, there are more than sufficient profits to distribute



(a) $\alpha = 1$, $v = 22$.



(b) $\alpha = 3$, $v = 500$.

Figure 4: The highest sustainable price $\hat{p}^{\star}(\bar{\kappa}; \delta)$ and the lower bound $\hat{\delta}$ as a function of the degree of heterogeneity ϵ. Here, $c(s, m) = \frac{s}{\alpha}\left(\frac{s}{m}\right)^{\alpha}$, $\delta = \frac{3}{4}$, and there are 16 firms; half of the firms have productive capacity $\frac{1}{16} + \epsilon$, and half of the firms have productive capacity $\frac{1}{16} - \epsilon$.

among firms so that each firm's per-period profits are large enough to sustain v. However, once the small firms are small enough, each small firm's cost of lone production rises above v, and these firms could receive 0 profits; thus $\hat{F}$ is the full set of firms when ϵ is small enough, i.e., when ϵ is small enough that $c\Big(1, \frac{1}{16} - \epsilon\Big) \leq v$, and $\hat{F}$ is the set of large firms otherwise.

As heterogeneity increases even further, we enter the right blue-line region. Here, a further increase in the size of each large firm continues to lower each large firm's cost of engaging in lone production; but now, there are not sufficient profits to distribute among the eight large firms to sustain v, even while each small firm receives 0 profits. Further increases in ϵ further increase the productive capacity of each of the large firms, decreasing the average cost of lone production across the eight large firms, i.e., $\hat{\varphi} \sum_{f \in \hat{F}} c\Big(1, \kappa^f\Big)$.

The right panel of Figure 4a plots the discount factor lower bound $\hat{\hat{\delta}}(\kappa;\delta)$ required by Theorem D.1. Since $\delta = \frac{3}{4}$, the discount factor is sufficiently high in all cases described above.

The left panel of Figure 4b shows the highest sustainable price as a function of ϵ and considers parameters so that each of the three scenarios of Theorem D.1 holds for some degree of heterogeneity. In the green-line region, firms are relatively homogenous, and the cost of lone production for each firm exceeds v.[71] In the red-line region, each of the eight large firms can now produce alone for less than v; however, there are still sufficient profits to distribute among firms to sustain v. Finally, in the blue-line region, each of the eight large firms is now large enough that v can no longer be sustained. The right panel of Figure 4b again plots the discount factor lower bound $\hat{\hat{\delta}}(\kappa;\delta)$ required by Theorem D.1 and shows that $\delta = \frac{3}{4}$ is sufficiently high.

The construction of the equilibrium that sustains $\hat{p}^{\star}(\kappa;\delta)$ is similar to that of the homogenous case. In the cooperation phase of our constructed equilibrium, each firm submits a bid of $\hat{p}^{\star}(\kappa;\delta)$. However, the amount of surplus received by each firm now depends on that firm's productive capacity. Larger firms, i.e., firms with a larger productive capacity, receive a greater share of surplus, as the cost of lone production is lower for a larger firm. After a price deviation, if syndication offers are insufficient and every non-leading firm rejects the price deviator's offer, then play enters a collusive punishment phase. The price in this collusive punishment phase is given by $\min\{c(1,\kappa^{\max}), v\}$. This ensures that no firm has an incentive to deviate and engage in lone production (as the cost of lone production will be at least the price). Finally, there is also a Bertrand reversion phase, in which the price is the cost of efficient joint production $c(1,k)$. Play enters this stage whenever any firm deviates with respect to accepting or rejecting offers of syndication. The restriction on the discount factor $\hat{\hat{\delta}}(\kappa;\delta)$ ensures that undercutting on price and recruiting a syndicate is not profitable—i.e., that the binding constraint on the highest sustainable price remains the profits available from price-deviating followed by lone production.

[71]Note that lone production is very costly in Figure 4b due to the high curvature of the cost function.

D.2 Market Entry

We now consider the effect of entry by a small firm on the highest sustainable price. When a firm enters the market, there are three possible effects: First, it may become easier for a price deviator to form a syndicate, making collusion more difficult. However, when the discount factor is high enough, a price-deviator will find forming a syndicate more costly than engaging in lone production, so this effect does not alter the highest sustainable price. Second, the new entrant may itself price-deviate and engage in lone production; this may make collusion more difficult. But, for a small enough entrant, the cost of lone production is higher than the highest sustainable price when the entrant is not present, and so the entrant will not price-deviate and engage in lone production.[72] Third, the additional productive capacity of the entrant reduces the cost of joint production, which makes collusion at the current price more profitable. This last effect always has bite, and so entry by a small enough entrant raises the highest sustainable price.

Proposition D.1. *If syndication offers specify both production shares and fees, $\delta > \hat{\delta}(\kappa; \delta)$, $\hat{p}^{\star}(\kappa; \delta) < v$, and $\lim_{m \to 0} c(s, m) = \infty$ for all $s > 0$, then there exists an $\epsilon > 0$ such that entry by a firm f with productive capacity $\kappa^f < \epsilon$ will increase the highest sustainable price, i.e.,*

$$\hat{p}^{\star}((\kappa, \kappa^f); \delta) > \hat{p}^{\star}(\kappa; \delta).$$

Figure 5 depicts the highest sustainable price for a simple economy as a function of the size of the entrant. When no entrant is present, the highest sustainable price is 15; however, for small entrants, the highest sustainable price is (slightly) higher than 15. This happens because a sufficiently small entrant does not have the productive capacity to profitably undercut the collusive price and engage in lone production. Moreover, the entrant's capacity makes collusion more profitable for the incumbent firms, as it decreases the cost of joint production. This makes collusion relatively more attractive to the incumbent firms, compared

[72] Similarly, if entrants are unable to bid but instead can only participate in the syndicate, the highest sustainable price will increase after entry.

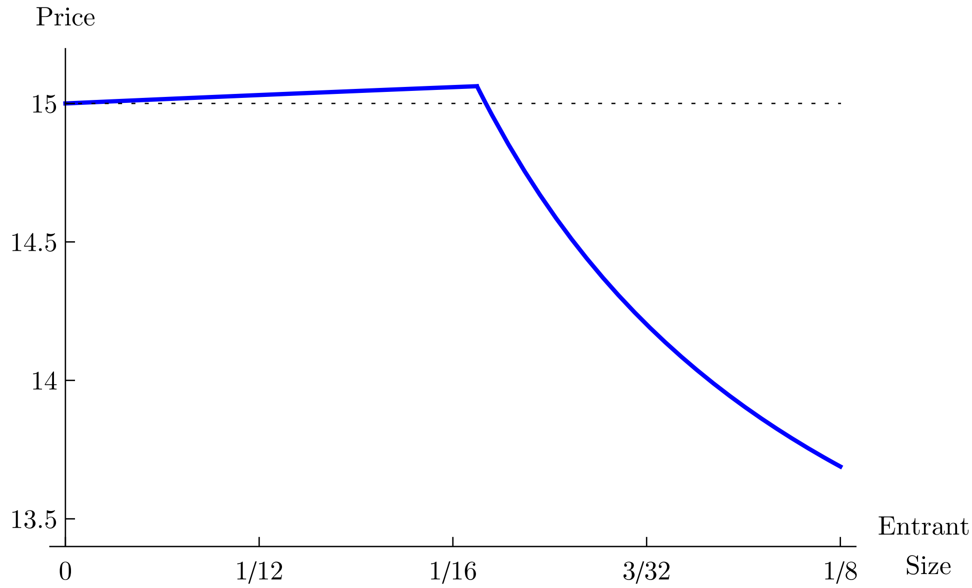


Figure 5: The highest sustainable price $\hat{p}^{\star}((\kappa, \kappa^f); \delta)$ as a function of entrant size κ^f. Here, $c(s, m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, $v = 25$, and there are 8 incumbent firms each with productive capacity $\frac{1}{8}$. The dashed line denotes the highest sustainable price without entry.

to price-deviating and engaging in lone production. Thus, entry by a sufficiently small firm will facilitate collusion as opposed to hampering it. In particular, our result here implies that the existence of a "competitive fringe" of small firms does not necessarily hamper the ability of larger firms to collude and sustain high prices.

However, for a sufficiently large entrant, collusion will become more difficult. An entrant with enough productive capacity can profitably undercut the collusive price by price-deviating and engaging in lone production; this occurs when κ^f becomes approximately $\frac{1}{16}$ in Figure 5. Thus, when the entrant has sufficient production capacity, some industry profits must be allocated to the entrant in order to make colluding a more rewarding option for the entrant than price-deviating and engaging in lone production. Allocating some profits to the entrant leaves fewer industry profits for the other firms, making collusion relatively less attractive to them. This makes collusion more difficult, reducing the highest sustainable price.

D.3 Proof of Theorem D.1

D.3.1 Constrained Profit Maximization

We first find the maximal price p subject to the constraints that each firm's total discounted profits are larger than its one-period profit from lone production at price p, each firm's profits are positive, and the price is no greater than v. To do this, we let r^f denote the share of profits allocated to firm f. Thus, we solve

$$\max_{p,r}\{p\} \tag{4}$$

subject to the constraints

$$\begin{aligned}
\frac{1}{1-\delta} r^f (p - c(1,k)) &\geq p - c\Big(1, \kappa^f\Big) && \text{for all } f \in F \\
r^f &\geq 0 && \text{for all } f \in F \\
p &\leq v \\
\sum_{f \in F} r^f &= 1.
\end{aligned}$$

We transform the problem given by (4) by letting $\pi^f = r^f (p - c(1,k))$ be the (per-period) profit for f, and so obtain the problem

$$\max_{\pi}\left\{\sum_{f \in F} \pi^f\right\} \tag{5}$$

subject to the constraints

$$\begin{aligned}
\frac{1}{1-\delta} \pi^f &\geq \left(\sum_{g \in F} \pi^g\right) + c(1,k) - c\Big(1, \kappa^f\Big) && \text{for all } f \in F \\
\pi^f &\geq 0 && \text{for all } f \in F \\
\sum_{f \in F} \pi^f &\leq v - c(1,k).
\end{aligned}$$

The first constraint is the *no lone deviation constraint*, the second constraint ensures that each firm's profits are non-negative, and the third constraint ensures that the price is no greater than v. This is a convex optimization problem; thus, by Theorem 7.16 of Sundaram (1996), if there exists a vector of continuation payoffs π within the constraint set and Lagrangian multipliers λ, μ, and ν that satisfy the Kuhn-Tucker conditions given π, i.e., for all $f \in F$,

$$1 + \frac{1}{1-\delta}\lambda^f - \sum_{g \in F} \lambda^g + \mu^f - \nu = 0 \tag{6}$$

$$\lambda^f \geq 0 \text{ and } \lambda^f \left(\frac{1}{1-\delta}\pi^f - \left(\sum_{g \in F} \pi^g \right) - c(1,k) + c\Big(1, \kappa^f\Big) \right) = 0 \tag{7}$$

$$\mu^f \geq 0 \text{ and } \mu^f \pi^f = 0 \tag{8}$$

$$\nu \geq 0 \text{ and } \nu \left(v - c(1,k) - \sum_{f \in F} \pi^f \right) = 0, \tag{9}$$

then π maximizes (5) subject to the constraints. To characterize the price and profit shares that solve these equations, we consider two cases:

First, suppose that $\sum_{f \in F} \max\Big\{v - c\Big(1, \kappa^f\Big), 0\Big\} > \frac{1}{1-\delta}(v - c(1,k))$. To find a solution to (6)–(9), we first find the unique $p \in (c(1,k), v)$ such that

$$\frac{1}{1-\delta}(p - c(1,k)) = \sum_{f \in F} \max\Big\{0, p - c\Big(1, \kappa^f\Big)\Big\}.$$

Intuitively, the left hand side of the equation is the industry profits produced, while the right hand side is the sum of the profit of each firm if it were to price deviate and engage in lone production (for those firms for which it is profitable to do so), which is the total amount of profits required to keep firms from deviating. The price that solves this equation exists and is unique: Both sides of the equation are 0 at $p = c(1,k)$, the slope of the left hand side (with respect to p) is always $\frac{1}{1-\delta}$, the slope of the right hand side around $p = c(1,k)$ is 0, both sides are continuous in p, and, at $p = v$, the right hand side is greater than the left hand side by assumption. Let $\hat{p}^\star$ be this unique solution, let $\hat{F} \equiv \Big\{f \in F : \hat{p}^\star > c\Big(1, \kappa^f\Big)\Big\}$ be the set of firms such that $\hat{p}^\star$ is greater than their cost of lone production, and let $\hat{\varphi} \equiv \frac{1}{|\hat{F}|}$. The solution to (6)–(9) is then given by

$$\pi^f = (1-\delta)\max\Big\{0, p - c\Big(1, \kappa^f\Big)\Big\}$$

$$\lambda^f = \begin{cases} \frac{(1-\delta)\hat{\varphi}}{1-\delta-\hat{\varphi}} & f \in \hat{F} \\ 0 & f \in F \smallsetminus \hat{F} \end{cases}$$

$$\mu^f = \begin{cases} 0 & f \in \hat{F} \\ \frac{\hat{\varphi}}{1-\delta-\hat{\varphi}} & f \in F \smallsetminus \hat{F}. \end{cases}$$

$$\nu = 0.$$

Finally, we solve for the $\hat{p}^\star$ implied by this solution to our convex program as

$$\hat{p}^\star = \frac{(1-\delta)\hat{\varphi}\sum_{f\in\hat{F}} c\Big(1,\kappa^f\Big) - \hat{\varphi}c(1,k)}{1-\delta-\hat{\varphi}}.$$

Second, suppose that $\sum_{f\in F}\max\Big\{v - c\Big(1,\kappa^f\Big), 0\Big\} \leq \frac{1}{1-\delta}(v - c(1,k))$. A solution to (6)–(9) is then given by[73]

$$\pi^f = (1-\delta)\max\Big\{0, v - c\Big(1,\kappa^f\Big)\Big\} + \varphi\left(v - c(1,k) - (1-\delta)\sum_{g\in F}\max\Big\{0, v - c\Big(1,\kappa^f\Big)\Big\}\right)$$

$$\lambda^f = 0$$

$$\mu^f = 0$$

$$\nu = 1.$$

D.3.2 Proving $\hat{p}^\star$ Is Sustainable

To show that $\hat{p}^\star(\kappa;\delta)$ is the highest sustainable price, we construct an equilibrium as follows:

[73] Note that in this case, the only binding constraint will be (9); thus, the per-period values π calculated here are not necessarily unique.

- There are three phases of equilibrium play:

  1. In the *cooperation phase*,

     – every firm submits the same bid $p = \hat{p}^{\star}(\kappa; \delta)$,

     – the short-lived buyer accepts one such offer of $\hat{p}^{\star}(\kappa; \delta)$, choosing each offer with equal probability,

     – every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi^g, \kappa^g) + \pi^g$ to each non-leading firm ℓ for agreeing to perform φ^g of production, where $\varphi^g \equiv \frac{\kappa^g}{k}$,[74] and

     – every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

  2. In the *collusive punishment phase with continuation values* ψ,

     – every firm submits the same bid $q = \min\{c(1, \kappa^{\max}), v\}$,

     – the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,

     – every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi^g, \kappa^g) + \psi^g$ to every non-leading firm $g \in F \setminus \{\ell\}$ to join the syndicate, and

     – every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

  3. In the *Bertrand reversion phase*, firms play the Bertrand reversion Nash equilibrium.[75]

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers.

[74] Note that each firm performing φ^g of production is the lowest-cost (i.e., efficient) production plan.

[75] Here, in the Bertrand reversion Nash equilibrium, the syndicate leader offers every other firm $c(\varphi^g, \kappa^g)$ for agreeing to perform φ^g of the production.

- If so, future play depends on the sum over the non-leading firms of the (positive) difference between

  – the syndication fee w^g offered to each firm g and

  – the cost to firm g of production s^g

  in the syndication contracts offered by f, i.e., $\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \kappa^g))^+$.[76] Based on this sum, we categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

  Uniformly Low Offers: $\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \kappa^g))^+ = 0$**.** In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production. Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

  Insufficient Offers: $0 < \sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \kappa^g))^+ \leq \frac{\delta}{1-\delta}(q - c(1, k))$**.** In this case, absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{\left(w^h - c\left(s^h, \kappa^h\right)\right)^+}{\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \kappa^g))^+} (q - c(1, k)) & h \neq f \\ 0 & h = f. \end{cases}$$

  Sufficient Offers: $\sum_{g \in F \setminus \{f\}} (w^g - c(s^g, \kappa^g))^+ > \frac{\delta}{1-\delta}(q - c(1, k))$**.** In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq c\left(s^h, \kappa^h\right)$.

  Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

[76] Here, $(x)^+ \equiv \max\{0, x\}$.

It is immediate that the conjectured equilibrium delivers a price of $\hat{p}^{\star}(\kappa;\delta)$ in each period. We now verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium.

Responding to Syndication Offers

We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each non-leading firm to accept its syndication offer.[77] It is also immediate that, in the case of uniformly low offers, it is a best response for each non-leading firm to reject its syndication offer.[78] Finally, it is immediate that, in the case of sufficient offers, each non-leading firm plays a best response; each non-leading firm only accepts its syndication offer if accepting provides a non-negative payoff in this period, and play continues to the Bertrand reversion phase regardless of the firm's actions.

To show that, in the case of insufficient offers, it is a best response for each non-leading firm to reject the offer of syndication, we calculate the total payoff for h from accepting the offer as

$$w^h - c\Big(s^h, \kappa^h\Big),$$

as play reverts to the Bertrand reversion phase if h accepts the offer (even if other firms reject their syndication offers). Meanwhile, the total payoff for h in the continuation game from rejecting its syndication offer is

$$\begin{aligned}\frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta}\left(\frac{\Big(w^h - c\Big(s^h, \kappa^h\Big)\Big)^+}{\sum_{g\in F\smallsetminus\{f\}}(w^g - c(s^g, \kappa^g))^+}(q - c(1,k))\right)\\ &\geq w^h - c\Big(s^h, \kappa^h\Big),\end{aligned}$$

[77]This follows as each syndication offer provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, in which the firm's future payoffs are 0.

[78]This follows as each syndication offer provides the firm with non-positive surplus and play continues to the Bertrand reversion phase regardless of the firm's actions.

where the inequality follows from the fact that $\sum_{g\in F\smallsetminus\{f\}}(w^g - c(s^g,\kappa^g))^+ \leq \frac{\delta}{1-\delta}(q - c(1,k))$, as we are in the insufficient offers case. Thus, it is a best response for every non-leading firm to reject its syndication offer in the insufficient offers case.

Responding to Price Offers

It is immediate that each short-lived buyer b_t is acting optimally as b_t always chooses one of the lowest price offers less than or equal to its reservation price v.

Deviating on Price or Syndication Offers in the Collusive Punishment Phase

We now verify that, during a collusive punishment phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most q from the buyer. Thus, firm f's profit in-period is at most $q - c\Big(1,\kappa^f\Big) \leq c(1,\kappa^{\max}) - c\Big(1,\kappa^f\Big) \leq 0$ as $q = \min\{v, c(1,\kappa^{\max})\}$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most 0. On the other hand, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating; consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during a collusive punishment phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with dynamic compensation totaling at least $\frac{\delta}{1-\delta}(q - c(1,k))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{q}_{\text{Price}} - \underbrace{c(1,k)}_{\substack{\text{Total cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,k))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}} = \left(1 - \frac{\delta}{1-\delta}\right)(q - c(1,k)) \leq 0,$$

where the last inequality follows as $\delta \geq \frac{1}{2}$. In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to 0. By contrast, if firm f continues with equilibrium play, it receives a non-negative payoff. Thus, not deviating is a best response for firm f.

Deviating on Price or Syndication Offers in the Cooperation Phase

Finally, we verify that, during the cooperation phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most $\hat{p}^\star(\kappa;\delta)$ from the buyer. Thus, firm f's profit in-period is at most $\hat{p}^\star(\kappa;\delta) - c\Big(1,\kappa^f\Big)$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most $\hat{p}^\star(\kappa;\delta) - c\Big(1,\kappa^f\Big)$. On the other hand, firm f enjoys profits each period of $r^f(\hat{p}^\star(\kappa;\delta) - c(1,k))$ by not deviating. Consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers so long as

$$\frac{1}{1-\delta} r^f(\hat{p}^\star(\kappa;\delta) - c(1,k)) \geq \hat{p}^\star(\kappa;\delta) - c\Big(1,\kappa^f\Big);$$

but this constraint is satisfied by the construction of $\hat{p}^\star(\kappa;\delta)$—see (4).

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during the cooperation phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with dynamic compensation totaling at least $\frac{\delta}{1-\delta}(q - c(1,k))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{\hat{p}^\star(\kappa;\delta)}_{\text{Price}} - \underbrace{c(1,k)}_{\substack{\text{Total cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,k))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}}. \tag{10}$$

In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to that given by (10). By contrast, if firm f continues with equilibrium play, firm f enjoys profits each period of $r^f(\hat{p}^\star(\kappa;\delta) - c(1,k))$. Consequently, it is not profitable for f to deviate and make sufficient offers so long as

$$\frac{1}{1-\delta} r^f(\hat{p}^\star(\kappa;\delta) - c(1,k)) \geq \hat{p}^\star(\kappa;\delta) - c(1,k) - \frac{\delta}{1-\delta}(q - c(1,k)).$$

Note that, for a small enough firm f, we could have $r^f = 0$. Thus, we must have δ large enough so that

$$0 \geq \hat{p}^\star(\kappa;\delta) - c(1,k) - \frac{\delta}{1-\delta}(q - c(1,k)).$$

Thus, solving for δ, we have

$$\delta \geq \frac{\hat{p}^\star(\kappa;\delta) - c(1,k)}{(\hat{p}^\star(\kappa;\delta) - c(1,k)) + (q - c(1,k))},$$

which will be satisfied since $q = \min\{c(1,\kappa^{\max}), \hat{p}^\star\}$.

Thus, for $\delta \geq \hat{\delta}(\kappa;\delta)$, $\hat{p}^\star(\kappa;\delta)$ can be sustained.

Maximality of $\hat{p}^\star(\kappa;\delta)$

It now remains to show that no price higher than $\hat{p}^\star(\kappa;\delta)$ can be sustained. There are two cases to consider, depending on whether $\hat{p}^\star(\kappa;\delta) = v$ or $\hat{p}^\star(\kappa;\delta) < v$: In the former case, no price greater than $\hat{p}^\star(\kappa;\delta) = v$ can be sustained as no buyer will accept an offer higher than v.

It is also immediate that we can not construct an equilibrium with a price higher than $\hat{p}^\star(\kappa;\delta) = \frac{(1-\delta)\hat{\varphi}\sum_{f\in\hat{F}} c\left(1,\kappa^f\right) - \hat{\varphi}c(1,k)}{1-\delta-\hat{\varphi}}$, since, by construction, under any such price some firm will have an incentive to slightly underprice and engage in lone production.

D.4 Proof of Proposition D.1

Let ϵ be small enough so that $c(1, \epsilon) > v$. Note that such an ϵ must exist, as $c(1, \epsilon) \to \infty$ as $\epsilon \to 0$. Solving for the highest sustainable price when f is present, i.e., solving the problem given in (4), we obtain

$$\hat{p}^\star((\kappa, \kappa^f); \delta) = \min\left\{ \frac{(1-\delta)\hat{\varphi} \sum_{f \in \hat{F}} c\left(1, \kappa^f\right) - \hat{\varphi} c\left(1, k + \kappa^f\right)}{1 - \delta - \hat{\varphi}}, v \right\}.$$

Note as $c(1, \epsilon) > v > \hat{p}^\star((\kappa, \kappa^f); \delta)$, we have that $f \notin \hat{F}$. Thus,

$$\hat{p}^\star((\kappa, \kappa^f); \delta) - \hat{p}^\star(\kappa; \delta) = \min\left\{ \hat{\varphi} \frac{c(1, k) - c\left(1, k + \kappa^f\right)}{1 - \delta - \hat{\varphi}}, v - \hat{p}^\star(\kappa; \delta) \right\} > 0.$$

Finally, we need that $\delta > \hat{\delta}((\kappa, \kappa^f); \delta)$; this might not be true for the ϵ we have worked with so far. But as $\hat{\delta}((\kappa, \kappa^f); \delta)$ is continuous in κ^f and $\delta > \hat{\delta}(\kappa; \delta)$ we have that $\delta > \hat{\delta}((\kappa, \kappa^f); \delta)$ for small enough ϵ—so if necessary, we can take ϵ still smaller (which preserves $c(1, \epsilon) > v$), proving the result.