# Taxation in Matching Markets*

Sonia Jaffe‡ (Job Market Paper) Scott Duke Kominers§

October 25, 2014

**Abstract**

What is the impact of taxation in matching markets? In matching markets, because agents have heterogeneous preferences over potential partners, welfare depends on which agents are matched to each other in equilibrium. Taxes in matching markets can generate inefficiency by changing who is matched to whom, even if the number of workers at each firm is unaffected. For markets in which workers refuse to match without a positive wage, higher taxes decrease match efficiency. However, in marriage markets or student–college matching markets, where transfers may flow in either direction, raising taxes may increase match efficiency. Simulations show that, in matching markets, calculations of deadweight loss based on the change in taxable income can be substantially biased in either direction.

*JEL classification:* H20, J24, C78, D13.

*Keywords*: Matching, taxation, efficiency, distortion.

*The authors appreciate the helpful comments of Gary Becker, Pierre-André Chiappori, Raj Chetty, David Cutler, Arnaud Dupuy, Steven Durlauf, Alexander Frankel, Roland Fryer, Alfred Galichon, Edward Glaeser, Jerry Green, Lars Hansen, John Hatfield, James Heckman, Nathaniel Hendren, Sabrina Howell, Stephanie Hurder, Adam Jaffe, Peter Klibanoff, Fuhito Kojima, David Laibson, Steven Levitt, Robert McCann, Stephen Morris, Kevin Murphy, Yusuke Narita, Charles Nathanson, Derek Neal, Alexandru Nichifor, Kathryn Peters, Philip Reny, Alvin Roth, Larry Samuelson, Florian Scheuer, James Schummer, Robert Shimer, Aloysius Siow, Tayfun Sönmez, Rakesh Vohra, E. Glen Weyl, and seminar and conference participants at the Becker Friedman Institute, the Fields Institute, Harvard University, Northwestern University, the Paris School of Economics, the Rotman School of Management, and the University of Wisconsin-Madison.

‡Department of Economics, Harvard University. Corresponding author (sjaffe@fas.harvard.edu). Jaffe gratefully acknowledges the support of a NSF Graduate Research Fellowship and a Terence M. Considine Fellowship in Law and Economics, as well as the hospitality of the Becker Friedman Institute for Research in Economics at the University of Chicago.

§Society of Fellows, Harvard University. Kominers is a Research Scientist at the Harvard Program for Evolutionary Dynamics and an Associate of the Harvard Center for Research on Computation and Society. He gratefully acknowledges the support of NSF grant CCF-1216095, a NSF Graduate Research Fellowship, a Terence M. Considine Fellowship in Law and Economics, a Yahoo! Key Scientific Challenges Fellowship, and an AMS–Simons Travel Grant. Much of the work on this paper was conducted while Kominers was a Research Scholar at the Becker Friedman Institute for Research in Economics at the University of Chicago.

# 1 Introduction

In many labor markets, there is not only vertical heterogeneity – some firms (and workers) are better than others – but horizontal heterogeneity – workers disagree about the desirability of different jobs and firms disagree about the desirability of different workers. Models of taxation that allow for worker heterogeneity and different types of work typically reduce heterogeneity in firm preferences to heterogeneity in wages – that is, each worker is paid according to his or her productivity in each job. However, when preferences are heterogeneous, firms may keep some of the productivity surplus – if a worker is more productive at one firm than at any other, that firm need not pay the worker his full productivity in equilibrium.[1]

In this paper, we analyze the impact of taxation on matching in markets with flexible preference heterogeneity. In these markets, match efficiency depends crucially on the assignment of agents to partners – not just on the set of agents who are matched in equilibrium. Consequently, a classical economic intuition holds, but with a caveat: raising taxes always increases equilibrium deadweight loss in markets where agents on one side of the market do not match unless they receive positive wages; however, raising taxes can decrease the deadweight loss in fully general matching markets.[2] There can be deadweight loss on the allocative margin – inefficiencies in the assignment of match partners – even if there are neither intensive nor extensive margin effects. Moreover, in the presence of two-sided preference heterogeneity, the change in taxable income is not a sufficient indicator of the welfare effect of taxation.

In the framework we develop, agents have heterogeneous rankings of potential match partners and may make transfer payments to their partners. Transfers may be "taxed," causing some of each payment to be taken from the agents.[3] In the case of a proportional tax $\tau$, an agent receives fraction $(1-\tau)$ of the amount his partner gives up (see Section 3);[4]. Taxation lowers the value of transfers, causing agents to prefer match partners that provide higher individual-specific match utilities over those offering higher transfers; for example, with taxation, a worker may switch to a firm he happens to enjoy, but where he is less productive. The tax reduces the firms' ability to compensate workers for the disutility of

[1]This is true even if firms are price-takers. The idiosyncratically high productivity at a firm means that firm's presence increases the surplus in the market; the firm gets to keep some of the marginal surplus.

[2]Most labor markets have wages flowing from firms to workers, though there may be internships that workers would pay to get. There are other, more balanced, matching markets where it may be more reasonable to think of transfers flowing in both directions. Most students pay for college, but a few are given living-stipends or free room and board in order to induce them to attend.

[3]We do not explicitly model the central authority that collects the tax. Our welfare analysis focuses on total match utility, implicitly assuming that the social value of tax revenue equals the private value.

[4]In Section 4 we look at lump sum transfers, where a fixed amount $f$ is subtracted from (non-zero) transfers

jobs where they are more productive, thereby distorting away from efficient matching.

The *matching distortion* we identify differs from the well-known effects of taxation on intensive and extensive labor supply; it affects the allocation of workers to firms without necessarily changing the provision of labor and it is not fully captured by the elasticity of taxable income. Also, matching distortions arise even in markets with frictionless search, and thus differ from the well-known effects of search costs on matching efficiency and of taxation on search behavior.[5]

Although our results are presented in the language of labor markets, they also have implications for other matching markets. Some transfers may be non-monetary and therefore may not be valued equally by givers and receivers: colleges may offer free housing to scholarship students, which may cost them more to provide than students' willingness to pay. Marriage markets also often have in-kind transfers: it may be the case that the utility a woman gives up by washing the dishes is greater than the utility her husband receives from her doing so.[6] Taxation can be reinterpreted as representing the frictions or loss on in-kind transfers. Because in college admissions and marriage markets positive transfers may flow in both directions, our non-monotonicity results indicate that it is hard to predict the efficiency response to a reduction in transfer frictions.

Of the vast literature on taxation, our work is most closely related to the research on the effect of taxation on workers' occupational choices (e.g., Parker (2003); Sheshinski (2003); Powell and Shan (2012); Lockwood et al. (2013)). However, this prior work only reflects part of the matching distortion because it does not model the two-sidedness of the market. If workers and firms *both* have heterogeneous preferences over match partners, then matching distortions can reduce productivity even without causing an aggregate shift in workers from one firm (or industry) to another.

Our approach is also related to the literature on taxation in Roy models (e.g., Rothschild and Scheuer (2012); Boadway et al. (1991)). The utility that a manager or firm in our model derives from a worker could be thought of the productivity of the worker in that firm or sector. However, most Roy models assume that workers earn their marginal product. Explicitly modeling firms allows for the possibility of taxation affecting the share of output that workers receive as wages.

Our model of matching with imperfect transfers provides a link between the canonical

[5]See, for example, the work of Blundell et al. (1998) and Saez (2004) on how taxation impacts the intensive margin, Meyer (2002) and Saez (2002) on how taxation impacts the extensive margin, Mortensen and Pissarides (2001) andBoone and Bovenberg (2002) on search costs, and Gentry and Hubbard (2004) and Holzner and Launov (2012) on taxation and search.

[6]A similar idea is modeled by Arcidiacono et al. (2011), who treat sexual activity as an imperfect transfer from women to men in the context of adolescent relationships.

models of matching with and without transfers: Absent taxation, our framework is equivalent to matching with perfect transfers (e.g., Koopmans and Beckmann (1957); Shapley and Shubik (1971); Becker (1974)); under 100% taxation, it corresponds to the standard model of matching without transfers (e.g., Gale and Shapley (1962); Roth (1982)). Thus, the intermediate tax levels we consider introduce a continuum of models between the two existing, well-studied extremes.

While prior work has analyzed frameworks that can embed our intermediate transfer models (Crawford and Knoer (1981); Kelso and Crawford (1982); Quinzii (1984); Hatfield and Milgrom (2005)), it has focused on the structure of the sets of stable outcomes *within* (fixed) models and has not looked at how the efficiency of stable outcomes changes *across* transfer models. It is therefore unable to analyze the effect of taxation. Legros and Newman (2007) do examine outcome changes across transfer models, but they use one-dimensional agent types and therefore have limited preference heterogeneity.

The remainder of the paper is organized as follows: Section 2 introduces our general model. Sections 3 and 4 analyze the cases of proportional and lump sum taxation, respectively. Section 5 discusses structural properties common to both models. Section 6 concludes. All proofs are presented in the Appendix.

# 2 General Model

Before introducing our models of taxation, we describe our underlying matching framework.

We study a two-sided, many-to-one matching market with fully heterogeneous preferences. We refer to agents on one side of the market as *managers*, denoted $m \in M$; we refer to agents on the other side *workers*, denoted $w \in W$. Our notation and language are also consistent with modeling marriage markets.

Each agent $i \in M \cup W$ derives utility from being matched to agents on the other side of the market. We denote these *match utilities* by $\alpha_m^Y$ and $\gamma_w^m$, with $\alpha_m^Y$ denoting the utility $m \in M$ obtains from matching with the set of workers $Y \subseteq W$ and $\gamma_w^m$ denoting the utility $w \in W$ obtains from matching with manager $m \in M$. Without loss of generality, we normalize the utility of being unmatched (an agent's reservation value) to 0, setting $\alpha_m^m = \gamma_w^w = 0$ for all $m \in M$ and $w \in W$. In the labor market context, $\alpha_m^Y$ may be the productivity of the set of workers $Y$ when employed by manager $m$ and $\gamma_w^m$ may be the utility or disutility worker $w$ gets from working for $m$.[7]

[7]Although it may seem that $\alpha_m^Y$ should be positive and $\gamma_w^m$ should be negative, for our general analysis we do not make sign assumptions. That is, we allow for the possibility of highly demanded internships and for counterproductive employees.

Note that it is possible for workers to disagree about the relative desirabilities of potential managers and for managers to disagree about the relative values of potential workers. We impose no structure on workers' match utilities and only impose enough structure on managers' preferences to ensure the existence of equilibria. For example, the match utilities could be random draws or may result from an underlying utility function in which agents have multi-dimensional types and preferences. To ensure existence, we assume that managers' preferences satisfy the standard Kelso and Crawford (1982)/Hatfield and Milgrom (2005) *substitutability* condition: the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.[8]

A *matching* $\mu$ is an assignment of agents such that each manager is either matched to himself (*unmatched*) or matched to a set of workers who are matched to him. Denoting the power set of $W$ by $\wp(W)$, a matching is then a mapping $\mu$ such that

$$\mu(m) \in (\wp(W) \cup \{m\}) \quad \forall m \in M,$$
$$\mu(w) \in (M \cup \{w\}) \quad \forall w \in W,$$

with $w \in \mu(m)$ if and only if $\mu(w) = m$.

We allow for the possibility of (at least partial) transfers between matched agents. We denote the transfer from $m$ to $w$ by $t^{m \to w} \in \mathbb{R}$; if $m$ receives a positive transfer from $w$, then $t^{m \to w} < 0$. A transfer vector $t$ identifies (prospective) transfers between all manager–worker pairs, not just between those pairs that are matched. We also include in the vector $t$ "transfers" $t^{i \to i}$ for all agents $i \in M \cup W$, with the understanding that $t^{i \to i} = 0$. For notational convenience, we denote by $t^{m \to Y}$ the total transfer from manager $m$ to workers in $Y$:

$$t^{m \to Y} \equiv \sum_{w \in Y} t^{m \to w}.$$

In the presence of taxation, a worker might not receive an amount equal to that which his match partner gives up; in general, a (weakly increasing) *transfer function* $\xi(\cdot)$ converts managers' transfer payments into the amounts that workers receive, post-tax, with $\xi(t^{m \to w}) \leq t^{m \to w}$. For all our transfer functions, we use the convention that $\xi(t^{w \to w}) = 0$ for all $w \in W$. With these conventions, we have the following lemma.

**Lemma 1.** *For a given matching $\mu$ and transfer vector $t$, the sum of transfers managers*

[8]Substitutability plays no role in our analysis other than ensuring, through appeal to previous work (Kelso and Crawford (1982)), that equilibria exist. Thus, we leave the formal discussion of the substitutability condition to the Appendix.

*pay to their match partners equals the sum of the transfers paid by workers' match partners,*

$$\sum_{m \in M} t^{m \to \mu(m)} = \sum_{m \in M} \sum_{w \in \mu(m)} t^{m \to w} = \sum_{w \in W} t^{\mu(w) \to w} \leq \sum_{w \in W} \xi(t^{\mu(w) \to w}). \tag{1}$$

An *arrangement* $[\mu; t]$ consists of a matching and a transfer vector.[9] We assume that agent utility is quasi-linear in transfers and that agents only care about their own match partner(s). With these assumptions, the *utility* values of arrangement $[\mu; t]$ for manager $m \in M$ and worker $w \in W$ are

$$u^m([\mu; t]) \equiv \alpha_m^{\mu(m)} - t^{m \to \mu(m)},$$
$$u^w([\mu; t]) \equiv \gamma_w^{\mu(w)} + \xi(t^{\mu(w) \to w}).$$

Note that the both the match utilities and the transfers may be either positive or negative. The utility of a worker $w \in W$ depends on the transfer function $\xi(\cdot)$.

Our analysis focuses on the arrangements that are stable, in the sense that no agent wants to deviate.

**Definition.** An arrangement $[\mu; t]$ is *stable given transfer function* $\xi(\cdot)$ if the following conditions hold:

1. Each agent (weakly) prefers his assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad \forall i \in M \cup W.$$

2. Each manager (weakly) prefers his assigned match partners (with the corresponding transfers) to any alternative set of workers (with the corresponding transfers), that is,

$$u^m([\mu; t]) = \alpha_m^{\mu(m)} - t^{m \to \mu(m)} \geq \alpha_m^Y - t^{m \to Y}, \qquad \forall m \in M \text{ and } Y \subseteq W;$$

   and each worker (weakly) prefers his assigned match partner (with the corresponding transfer) to any alternative manager (with the corresponding transfer), that is,

$$u^w([\mu; t]) = \gamma_w^{\mu(w)} + \xi(t^{\mu(w) \to w}) \geq \gamma_w^m + \xi(t^{m \to w}) \qquad \forall w \in W \text{ and } m \in M.$$

[9] Here were use the term "arrangement" instead of "outcome" for consistency with the matching literature (e.g., Hatfield et al. (2013)), which uses the latter term when the transfer vector only includes transfers between agents who are matched to each other.

A matching $\mu$ is *stable given transfer function* $\xi(\cdot)$ if there is some transfer vector $t$ such that the arrangement $[\mu; t]$ is stable given $\xi(\cdot)$; in this case $t$ is said to *support* $\mu$ (given $\xi(\cdot)$).

Arguments of Kelso and Crawford (1982) show that the stability concept we use is equivalent to the other standard stability concept of matching theory, which rules out the possibility of "blocks" in which groups of agents jointly deviate from the stable outcome (potentially adjusting transfers).[10] The assumption of substitutable preferences ensures that at least one stable arrangement always exists.[11]

In analyzing stable arrangements we focus on the *total match utility* of the match $\mu$, defined as

$$\mathfrak{M}(\mu) \equiv \sum_{m \in M} \alpha_m^{\mu(m)} + \sum_{w \in W} \gamma_w^{\mu(w)}.$$

We do not model the institution imposing the tax; as we focus on match utility, our analysis is most relevant to the case where the social value of tax revenue equals the private value.

**Definition.** We say that a matching $\hat{\mu}$ is *efficient* if it maximizes total match utility among all possible matchings, i.e. if $\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\mu)$ for all matchings $\mu$.[12]

Some of our analysis focuses on markets in which workers have nonpositive valuations for matching, so that they will only match if paid positive "wage" transfers. Formally, we say that a market is a *wage market* if

$$\gamma_w^m \leq 0 \tag{2}$$

for all $w \in W$ and $m \in M$; it is a *strictly positive wage market* if the inequality in (2) is *strict* for all $w \in W$ and $m \in M$. The existence of internships notwithstanding, most labor markets can be reasonably modeled as wage markets.

For simplicity, we set our illustrative examples in *one-to-one matching markets*, in which each manager matches to at most one worker. For such markets, we abuse notation slightly by only specifying match utilities for manager–worker pairs and writing $w$ in place of the set $\{w\}$ (e.g., $\alpha_m^{\{w\}}$ is denoted $\alpha_m^w$).

[10] Our stability concept is defined in terms of arrangements; the block-based definition is defined only in terms of a matching and the transfers *between matched partners*. Kelso and Crawford (1982) used the term *competitive equilibrium* for the former concept and used *the core* to refer to the latter.

[11] Results of Kelso and Crawford (1982) guarantee the existence of a stable arrangement in our framework. Details are provided in the Appendix.

[12] An alternative welfare measure would be *total agent utility*, i.e. total match utility minus total tax revenue. However, while government expenditures may not always be valued dollar-for-dollar, including government revenue in welfare is typically considered a better approximation than assigning it no value (Mas-Colell et al., 1995). Moreover, total agent utility depends on the transfer vector; as there are frequently many transfer vectors supporting a given stable match, total agent utility is not typically well-defined, even fixing a given stable match and/or tax function. The possibility of non-monotonicities can easily be shown to extend to agent utility.

# 3 Proportional Taxation

First we analyze proportional (linear) taxation systems, of the type used in some US states and dozens of countries around the world. These taxes take the form of a fixed percentage deduction of each agent's income. Formally, under proportional tax $\tau$, if an agent pays $p$, then his partner receives $(1-\tau)p$. The associated transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ is

$$\xi_\tau^{\mathsf{prop}}(t^{m\to w}) \equiv \begin{cases} (1-\tau)t^{m\to w} & t^{m\to w} \geq 0 \\ \frac{1}{(1-\tau)}t^{m\to w} & t^{m\to w} < 0. \end{cases}$$

Figure 1 illustrates the transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ for different tax rates $\tau$.

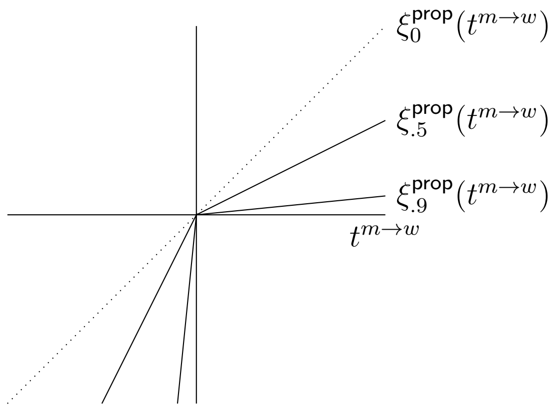



Figure 1: Transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$.

If an arrangement $[\mu; t]$ or matching $\mu$ is stable given $\xi_\tau^{\mathsf{prop}}(\cdot)$, then we say it is *stable under tax* $\tau$. We analyze how the set of stable matchings changes as $\tau$ decreases from 1 to 0.

The case $\tau = 1$ corresponds to the standard Gale and Shapley (1962) setting in which transfers are not allowed,[13] so inefficient matchings may be stable. When $\tau = 0$, by contrast, only efficient matchings are stable (see, e.g., Shapley and Shubik (1971); Hatfield et al. (2013)). Given this, one might expect that as the tax rate $\tau$ decreases, the match utilities of stable matchings should always (weakly) increase. Unfortunately, a simple example shows that this is not true in general.

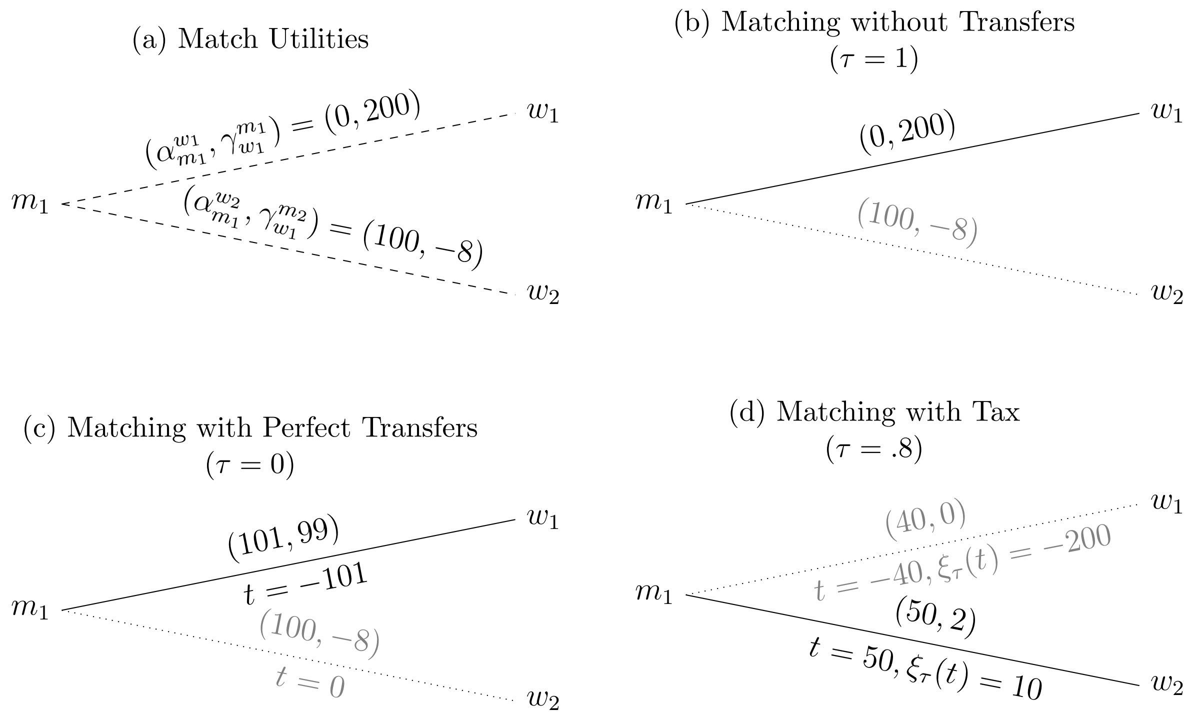



Figure 2: Example 1 – Non-monotonicity under a proportional tax on transfers.
Note: Utilities, net of transfers, are above the lines (manager's, worker's). Possible supporting transfers (when applicable) are below the lines. Solid lines indicate the stable matching.

## 3.1 Possible Inefficiencies of Tax-Reduction in General Markets

**Example 1.** Consider a one-to-one market with one manager, $M = \{m_1\}$, two workers, $W = \{w_1, w_2\}$, and match utilities as pictured in Figure 2a. Worker $w_1$ receives high utility from matching with $m_1$. Manager $m_1$ is indifferent towards worker $w_1$ and receives moderate utility from matching with $w_2$. Worker $w_2$ has a mild preference for being unmatched, rather than matching with $m_1$.

We can think of $w_1$ as an intern who would not be very productive in working for $m_1$, but would learn a lot; $w_2$ represents a normal worker, who is productive but does not like working. With this interpretation, the tax represents a proportional income tax – which $m_1$ must also pay if the intern $w_1$ bribes him in exchange for a job. Alternatively, we may interpret the example in a marriage context: $w_1$ is an unremarkable woman who really wants to get married; $w_2$ is a highly desirable woman who prefers to remain single; and $m_1$ is the last man on Earth. In that case, the tax reflects the extent to which it is difficult to transfer

[13]When $\tau = 1$, the set of stable matchings is the same as in the case that transfers are not allowed. The associated arrangements are not exactly the same, however, because the supporting transfer vectors need not be equal to 0. However, if $\mu$ is stable when $\tau = 1$, then there is a transfer vector $t$ supporting $\mu$ such that $t^{m \to w} = 0$ for all $m \in M$ and $w \in \mu(m)$; the arrangement $[\mu; t]$ therefore replicates the utilities that arise under $\mu$ when transfers are not allowed.

utility between individuals within a couple.

As illustrated in Figure 2b, when $\tau = 1$ (or when transfers are not allowed), the only stable matching $\hat{\mu}$ has $\hat{\mu}(m_1) = w_1$. This happens to be the efficient matching; therefore, it is also stable when $\tau = 0$, as shown in Figure 2c. This matching yields total match utility $\mathfrak{M}(\hat{\mu}) = 200$.

Figure 2d shows that for $\tau = .8$, an inefficient matching $\tilde{\mu}$, for which $\tilde{\mu}(m_1) = w_2$, is stable. This matching generates a total match utility $\mathfrak{M}(\tilde{\mu}) = 92$. Even if $w_1$ transfers 200 – his maximal utility of matching – to $m_1$, there is a transfer $m_1$ can offer to $w_2$ that is sufficient to attract $w_2$, while still providing $m_1$ more utility than he would obtain from matching with $w_1$ (and receiving $(1 - .8)(200) = 40$).

Not only is an inefficient matching stable under tax $\tau = .8$, but the efficient matching $\hat{\mu}$ is *not* stable under this tax. Indeed, the efficient matching $\hat{\mu}$ is unstable under any tax $\tau \in (.6, .9)$. For that range, $(100 - 200(1 - \tau))(1 - \tau) - 8 > 0$, so that the maximum $m_1$ can transfer to $w_2$ while still preferring $w_2$ to $w_1$ is sufficient to outweigh the disutility $w_2$ gets from matching to $m_1$.[14]

While Example 1 may appear quite specialized, simulations suggest that non-monotonicities in the total match utility of stable matches as a function of $\tau$ can be relatively common. We examine simulations of a one-to-one market with twenty agents on each side of the market and match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, $\tau$, from 0 to .99 in increments of .01. For each tax rate, we find the manager-optimal stable arrangement and calculate the total match utility.[15] Non-monotonicities in the total match utility of stable matchings appear in over half of the markets (55%).[16]

Figure 3 plots the total match utility as a function of the tax rate in ten randomly-selected simulation markets with non-monotonicities. These ten markets are fairly representative, in that they have relatively small losses from non-monotonicity, mostly occuring at high tax rates. Nevertheless the non-monotonicities in our simulation markets can be dramatic. Figure 4 presents a simulation market in which, just as in Example 1, the efficient matching is stable under full taxation ($\tau = 1$) but is unstable under a range of tax rates between 0

[14]Note that here total agent utility (match utility minus government revenue), like total match utility, can be non-monotonic. When $\tau = 1$, total agent utility is 200 (assuming they do not burn money). When $\tau = .8$ it is 52.

[15]If there are multiple stable arrangements, the manager-optimal arrangement is the one preferred by all managers. See Section 5 for a discussion of opposition of worker and manager interests when there are multiple stable arrangements.

[16]There may be additional non-monotonicities that we do not observe because we cannot vary $\tau$ continuously. However, the non-monotonicities we fail to observe necessarily occur over very small ranges of $\tau$, as we observe all non-monotonicities that persist over values of $\tau$ with a range of .01 or more.

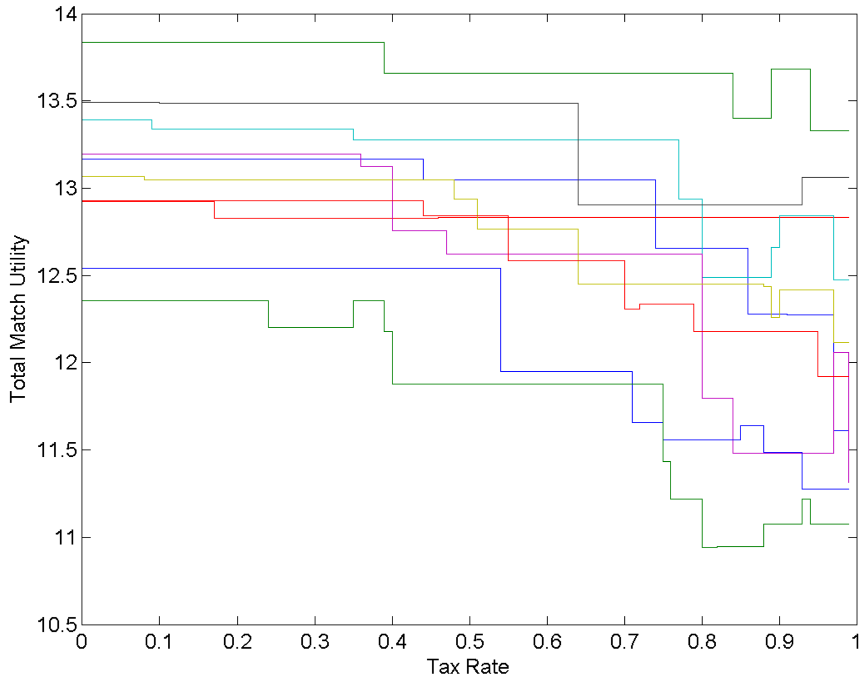



Figure 3: Total match utility of a stable match in ten simulated markets.

Note: The markets presented were randomly-selected from the set of simulated markets exhibiting non-monotonicities. Each market is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. For each tax rate total match utility is calculated for the manager-optimal stable arrangement.

and 1.

Table 1 summarizes the non-monotonocities arising in our simulations. Row 1 shows the fraction of markets that have non-monotonicities in a given tax rate range. While the majority of non-monotonicities occur at very high tax rates, 10% of our simulation markets have non-monotonicities at tax rates below 50%. Row 2 gives the (normalized) average size of the non-monotonicities in each tax rate range. Again, we see that non-monotonicities are most significant for high tax rates. Row 3 incorporates information on the persistence of non-monotonicities by computing the fraction of the deadweight loss from taxation that is due to a non-monotonicity. This is relatively high for lower tax rates because there is less total deadweight loss at those tax rates.

Overall, our simulations suggest non-monotonicities in the tax rate are not just artifacts of example selection. However, they also suggest that non-monotonicities are relatively rare at more realistic tax rates ($\tau \in [0, .5)$) and tend not to persist over large ranges of $\tau$.[17]

Although Example 1 and the simulations show that total match utility of stable matchings

[17] Increasing the sample size does not appear to decrease the frequency or importance of non-monotonicities.

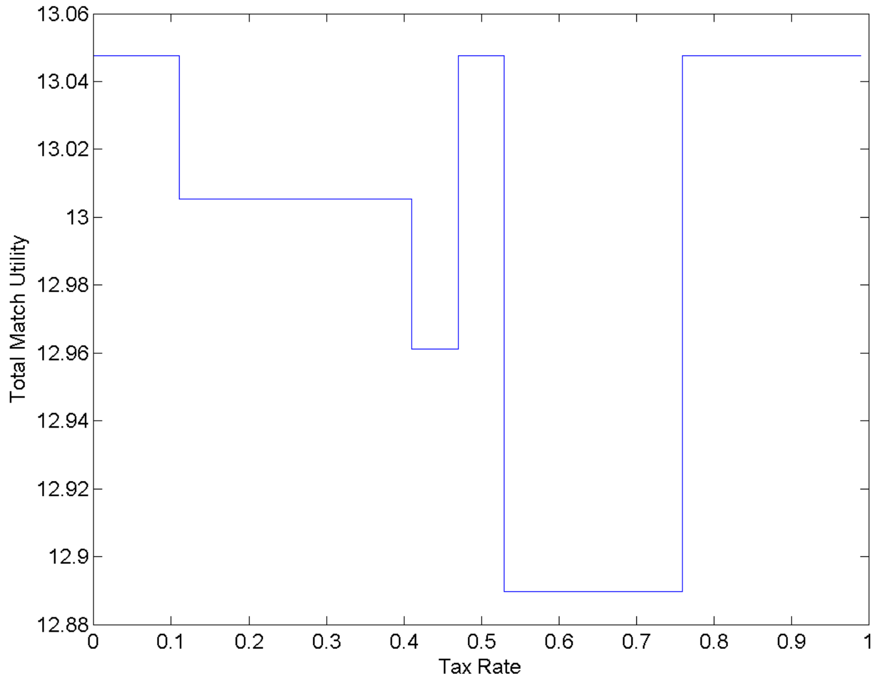



Figure 4: Total match utility of a stable match in a selected simulated market.

Note: The market pictured is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. For each tax rate total match utility is calculated for the manager-optimal stable arrangement.

may decrease when the tax rate falls, an arrangement that is stable under a tax rate $\hat{\tau}$ must improve the utility of at least one agent, relative to an arrangement that is stable under a tax rate $\tilde{\tau} > \hat{\tau}$.

**Proposition 1.** *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau}$, with $\tilde{\tau} > \hat{\tau}$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau}$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$).*[18]

To see the intuition behind Proposition 1, we consider the case in which $\tilde{\tau} = 1$ and choose $\tilde{t} = 0$: If $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$), then every manager $m \in M$ (weakly) prefers $\tilde{\mu}(m)$ to $\hat{\mu}(m)$ with the transfer $\hat{t}^{m \to \hat{\mu}(m)}$.[19] But then, because $[\hat{\mu}; \hat{t}]$

[18] We say that an arrangement $[\tilde{\mu}; \tilde{t}]$ under tax $\tilde{\tau}$ *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ if

$$\begin{aligned} \alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} &\geq \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} & \forall m \in M, \\ \gamma_w^{\tilde{\mu}(w)} + \xi_{\tilde{\tau}}^{\mathsf{prop}}(\tilde{t}^{\tilde{\mu}(w) \to w}) &\geq \gamma_w^{\hat{\mu}(w)} + \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\hat{\mu}(w) \to w}) & \forall w \in W, \end{aligned}$$

with strict inequality for some $i \in M \cup W$.

[19] To see this, we first note that under tax $\tilde{\tau} = 1$, an arrangement with transfers of 0 among match partners Pareto dominates any other arrangement associated to the same matching. Thus, the transfers between match partners under $[\tilde{\mu}; \tilde{t}]$ can be assumed to be 0. Then, the comparison between $[\tilde{\mu}; \tilde{t}]$ (under

Table 1: Summary of the non-monotonicities arising in simulated markets.

| | Range of $\tau$ | | | | |
|---|---|---|---|---|---|
| | $[0, .25)$ | $[.25, .5)$ | $[.5, .75)$ | $[.75, 1)$ | All $\tau$ |
| Fraction of markets with non-monotonicity | 0.006 | 0.088 | 0.190 | 0.394 | 0.548 |
| Avg size of non-monotonicity, as fraction of range | 0.021 | 0.066 | 0.111 | 0.140 | 0.120 |
| Fraction of deadweight loss from taxation due to non-monotonicity | 0.076 | 0.070 | 0.051 | 0.027 | 0.037 |

Note: The table summarizes 500 simulations of one-to-one matching markets with 20 agents on each side of the market. All agents' match utilities are independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We vary the tax rate, $\tau$, from 0 to .99 in increments of .01. For each tax rate, we find the manager-optimal stable arrangement and calculate the total match utility. Row 1 presents the fraction of markets that have non-monotonicities in a given tax rate range. Row 2 presents the average size of non-monotonicities within each range, normalized as a fraction of the (within-market) gap between the highest and lowest total stable match utilities calculated for any tax rate. Row 3 presents the average fraction of taxation deadweight loss that is due to non-monotonicity, across all markets. The deadweight loss from non-monotonicity is computed for each tax rate $\tau$ as the difference between the highest total match utility for a tax rate $\tilde{\tau} \geq \tau$ and the total match utility under tax rate $\tau$; it is divided by the total deadweight loss from taxation at tax rate $\tau$, which is computed as the difference in total match utility between the efficient matching and the matching stable under tax rate $\tau$.

is stable under tax $\hat{\tau}$,

$$\alpha_m^{\tilde{\mu}(m)} \overbrace{\geq}^{\text{Pareto}} \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \overbrace{\geq}^{\text{Stability}} \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)},$$

so every $m$ must be offering a weakly positive transfer to $\tilde{\mu}(m)$ under $\hat{t}$ (that is, $\hat{t}^{m \to \tilde{\mu}(m)} \geq 0$). An analogous argument shows that each worker $w \in W$ must be offering a weakly positive transfer to $\tilde{\mu}(w)$ under $\hat{t}$ (that is, $\xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w) \to w}) \leq 0$). Moreover, Pareto dominance implies that at least one manager or worker must be paying a *strictly* positive transfer. But then, that agent must pay a strictly positive transfer and receive a weakly positive transfer – impossible.

tax $\tilde{\tau} = 1$) and $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$) amounts to a comparison of agents' match utilities under $\tilde{\mu}$ and their total utilities under $[\hat{\mu}; \hat{t}]$.

## 3.2 Efficiency of Tax-Reduction in Wage Markets

The preceding discussion shows that in general markets, decreasing the tax rate on transfers may *decrease* the total match utility of stable matchings. Our next result shows that in wage markets, these non-monotonicities do not arise – decreasing the tax rate in a wage market always makes (weakly) more efficient matchings stable.[20]

In wage markets, payments flow from managers to workers; hence, any stable matching can be supported by a non-negative transfer vector.[21] Thus, the transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ takes the simpler form

$$\xi_\tau^{\mathsf{prop}}(t^{m\to w}) = (1-\tau)t^{m\to w} \geq 0.$$

As all positive transfers are paid from managers to workers, there cannot be a scenario in which, as in Example 1, when the tax is reduced, a manager can transfer enough to get a worker he prefers ($w_2$), but when the tax falls more, a different worker ($w_1$) can "buy back" the manager. Our next result shows that this intuition extends to wage markets more generally.

**Theorem 1.** *In a wage market with proportional taxation, a decrease in taxation (weakly) increases the total match utility of stable matchings. That is, if in a wage market, matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, matching $\hat{\mu}$ is stable under tax $\hat{\tau}$, and $\hat{\tau} < \tilde{\tau}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

To prove Theorem 1, we let $\hat{t} \geq 0$ and $\tilde{t} \geq 0$ be transfer vectors supporting $\hat{\mu}$ and $\tilde{\mu}$ respectively. The stability of $[\hat{\mu};\hat{t}]$ under tax $\hat{\tau}$ implies that

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{3}$$

$$\gamma_w^{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \gamma_w^{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w}. \tag{4}$$

[20] The non-monotonicities described in Section 3.1 arise from transfers flowing in both directions, either simultaneously or across equilibria. As transfers are an equilibrium phenomenon, requiring that transfers flow in one direction does not directly correspond to conditions on the primitives of the market. However, the wage market condition we use in Theorem 1 is a *sufficient* condition on primitives to guarantee that transfers to flow in one direction, and thus is sufficient to rule out non-monotonicity.

All the results in this section hold in any market where transfers always (across stable arrangements and tax rates) flow in one direction.

[21] There may be a supporting transfer vector where some off-path transfers (transfers between unmatched agents) are negative, but in that case there is always another supporting transfer vector that replaces those negative transfers with 0s. Our results only require the existence of a non-negative supporting transfer vector.

Summing (3) and (4) across agents, applying Lemma 1, and regrouping terms, we find that

$$\begin{aligned}\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m \in M} (\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}) + \sum_{w \in W} (\gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)}) \\ &\geq \hat{\tau} \sum_{m \in M} \left( \hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)} \right). \end{aligned} \tag{5}$$

Intuitively, since the tax change has a larger effect on larger transfers, if we had

$$\sum_{m \in M} \left( \hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)} \right) < 0,$$

then lowering the tax from $\tilde{\tau}$ to $\hat{\tau}$ would increase workers' relative preference for $\tilde{\mu}$ over $\hat{\mu}$. Since $\hat{\mu}$ is stable under the lower tax $\hat{\tau}$, the difference in (5) must thus be positive; this implies Theorem 1.

Theorem 1 shows that the non-monotonicities observed in fully general markets in Section 3.1 do not arise in wage markets. To gain insight into how quickly non-monotonicity disappears as a market's structure becomes closer to that of a wage market, we return to our simulation environment. We begin with simulations in a setting identical to that used in Section 3.1: one-to-one markets with all match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. We next consider one-to-one markets with match utilities slightly imbalanced across the market: managers' match utilities are independently and identically distributed according to a uniform distribution on $[-.45, .55]$, while workers' match utilities are independently and identically distributed according to a uniform distribution on $[-.55, .45]$. We repeat this process, adjusting the match utility means by .05 each time, to generate a series of markets ranging from our original symmetric markets to wage markets with manager utilities distributed uniformly on $[0, 1]$ and worker utilities distributed uniformly on $[-1, 0]$.[22] Figure 5 shows the how the fraction of markets with non-monotonocities in the match utility of the manager-optimal stable arrangement changes as the mean manager utility varies. Even fairly asymmetrical markets have moderate rates of non-monotonicities.

Although total match utility in wage markets increases as the tax is reduced, *individual* utility may be non-monotonic. For example, pursuant to a tax decrease, a manager $m$ may be made worse off because his match partner is now able to receive more from some other manger: In this circumstance, $m$ might lose his match partner to his competitor; even if $m$'s match is unchanged, his total utility may decrease because he is forced to increase his

[22]To reduce noise in the simulation process, we use a single set of 500 baseline markets and repeatedly shift each match utility by .05 in the appropriate direction.

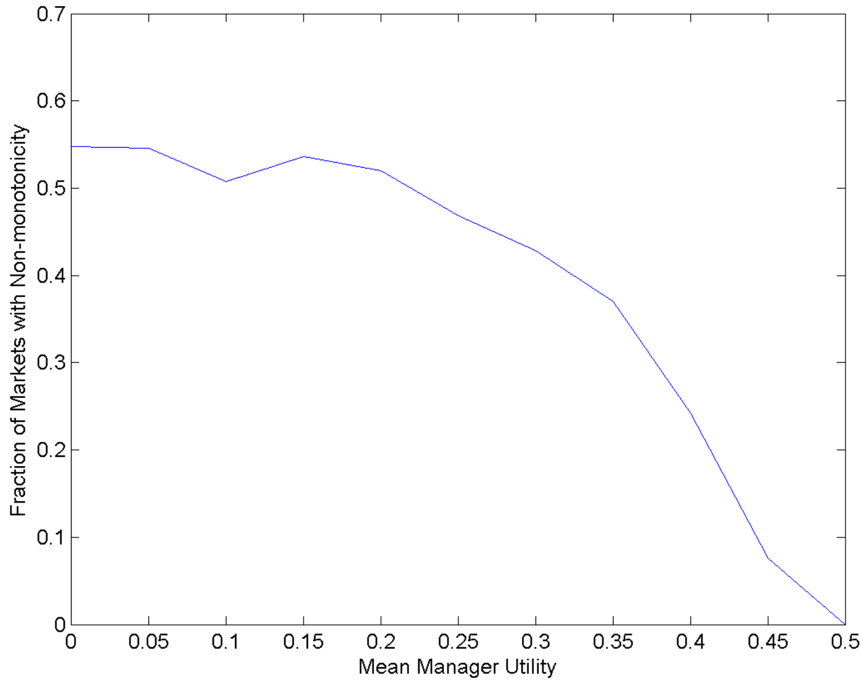



Figure 5: Fraction of simulated markets where the total match utility is non-monotonic in the tax rate, $\tau$.

Note: For each mean manger utility level, we report the fraction of the 500 simulated markets that have a non-monotonicity in total match utility of the manager-optimal arrangement as the tax rate increases from 0 to 1. All simulated markets are one-to-one and have 20 agents on each side of the market.

transfer to compensate for a competitor's increased offer.

Individual managers' match utilities *may* decrease with a decrease in the tax rate, but the sum of workers' match utilities *must* decrease.

**Proposition 2.** *In a wage market with proportional taxation, if a matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, and a matching $\hat{\mu}$ is stable under tax $\hat{\tau} < \tilde{\tau}$, then workers' aggregate match utility must be (weakly) higher under $\tilde{\mu}$ than under $\hat{\mu}$. That is,*

$$\sum_{w \in W} \gamma_w^{\tilde{\mu}(w)} \geq \sum_{w \in W} \gamma_w^{\hat{\mu}(w)}.$$

The logic is that in order for a less efficient match to be stable at the higher tax rate, it must be that workers prefer that match – and managers cannot lure workers to a more efficient match because of the high tax. As the tax rate decreases, managers' ability to make transfers to workers increases, and so the weight put on their match utilities relative to workers' match utilities increases. Absent taxation, stable matches maximizes the sum of match utilities with equal weight on managers and workers; under taxation, stable matches still maximize the sum of match utilities, but with different weights.

**Proposition 3.** *In a wage market with proportional taxation, if a matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, then $\tilde{\mu}$ is a matching that maximizes the sum of worker match utilities plus $(1-\tilde{\tau})$ times the sum of manager match utilities:*

$$\tilde{\mu} \in \arg\max_{\{\mu\}} \left[ (1-\tilde{\tau}) \sum_{m \in M} \alpha_m^{\mu(m)} + \sum_{w \in W} \gamma_w^{\mu(w)} \right].$$

Note that Propositions 2 and 3 do not imply that lower taxes necessarily make workers worse off: workers might receive transfers sufficiently high as to more than compensate for their lower match utilities.

Finally, we show that if two distinct matchings $\hat{\mu}$ and $\tilde{\mu}$ are both stable under tax $\tau$, then either managers and workers must disagree as to which matching is preferred, or both groups must be indifferent between the two matchings. This is a consequence of the following more general result.

**Proposition 4.** *In a wage market with proportional taxation, if two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable under tax $\tau$, then*

$$\sum_{w \in W} \left( \gamma_w^{\tilde{\mu}(w)} - \gamma_w^{\hat{\mu}(w)} \right) = (1-\tau) \sum_{m \in M} \left( \alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)} \right). \tag{6}$$

*Thus, if the managers are not indifferent in aggregate between $\tilde{\mu}$ and $\hat{\mu}$, then the only tax rate $\tau$ under which both $\tilde{\mu}$ and $\hat{\mu}$ can be stable is*

$$\tau = 1 + \frac{\sum_{w \in W} \left( \gamma_w^{\tilde{\mu}(w)} - \gamma_w^{\hat{\mu}(w)} \right)}{\sum_{m \in M} \left( \alpha_m^{\tilde{\mu}(m)} - \alpha_m^{\hat{\mu}(m)} \right)}. \tag{7}$$

For $\tau$ as defined in (7) to be less than 1, the fraction in (7) must be negative, so that that managers and workers in aggregate disagree about which matching they prefer.

In order for there to be multiple values of $\tau$ at which two given matchings are both stable, it must be that both managers and (following (6)) workers are indifferent between those two matchings.

**Corollary 1.** *In a wage market with proportional taxation, if there is more than one tax under which two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ both are stable, then $\mathfrak{M}(\tilde{\mu}) = \mathfrak{M}(\hat{\mu})$.*

Corollary 1 implies that for generic match utilities, there is at most one value of $\tau$ at which two matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable; in this case, since there are finitely many matchings, there is a unique stable matching under almost every tax $\tau$.

At a tax rate $\tau$ under which two distinct matches $\hat{\mu}$ and $\tilde{\mu}$ are stable, we can renormalize the utilities and use results from matching with transfers to draw conclusions about match utilities and revenue.[23] If we multiply all worker utilities and transfers by $\frac{1}{1-\tau}$, then workers' comparison of options is unchanged – and therefore the stable matches are unaffected – but the new utility function

$$u^w_\tau([\mu,t]) \equiv \frac{1}{1-\tau}u([\mu,t]) = \frac{1}{1-\tau}\gamma_w^{\mu(w)} + t^{\mu(w)\to w}$$

is quasi-linear. When worker and manger utilities are quasi-linear in the transfer, then results of Hatfield et al. (2013) show that if two matches, $\hat{\mu}$ and $\tilde{\mu}$ are stable, any transfer vector $t$ that supports one also supports the other. Moreover, for a given transfer vector, $t$, all agents are indifferent between the two arrangements:

$$\alpha_m^{\tilde{\mu}(m)} - t^{m\to\tilde{\mu}(m)} = \alpha_m^{\hat{\mu}(m)} - t^{m\to\hat{\mu}(m)}, \tag{8}$$

$$\frac{1}{1-\tau}\gamma_w^{\tilde{\mu}(w)} + t^{w\to\tilde{\mu}(w)} = \frac{1}{1-\tau}\gamma_w^{\hat{\mu}(w)} + t^{m\to\hat{\mu}(w)} \tag{9}$$

Multiplying (9) by $(1-\tau)$ we get:

$$\gamma_w^{\tilde{\mu}(w)} + (1-\tau)t^{w\to\tilde{\mu}(w)} = \gamma_w^{\hat{\mu}(w)} + (1-\tau)t^{m\to\hat{\mu}(w)}, \tag{10}$$

Summing (8) and (10) across agents gives the difference in total match utility:

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) = \tau\sum_{m\in M} t^{m\to\tilde{\mu}(m)} - \tau\sum_{m\in M} t^{m\to\hat{\mu}(m)}.$$

These results are summarized in the following proposition.

**Proposition 5.** *In a wage market, if two matches $\tilde{\mu}$ and $\hat{\mu}$ are both stable under tax rate $\tau$, then:*

1. *Any transfer vector that supports either $\tilde{\mu}$ or $\hat{\mu}$ also supports the other.*
2. *For any transfer vector $t$ that supports $\tilde{\mu}$ and $\hat{\mu}$ all agents are indifferent between $[\tilde{\mu}, t]$ and $[\hat{\mu}, t]$*

[23] This re-normalization, which was not in the original draft of this paper, was independently originated by Ismael Mourifie and Aloysius Siow, and Arnaud Dupuy and Alfred Galichon.

*3. The difference in total match utility between $\tilde{\mu}$ and $\hat{\mu}$ equals the difference in revenue $[\tilde{\mu}, t]$ and $[\hat{\mu}, t]$ for every transfer vector $t$ supporting $\tilde{\mu}$ and $\hat{\mu}$.*

Unfortunately, the third part of Proposition 5 – that changes in revenue sometimes correspond to changes in utility – is very limited. As the tax rate changes, transfers will change even when the underlying match does not change (so there is no change in total match utility). Also, even at the tax rate where multiple matches are stable, there may be multiple supporting transfer vectors and the revenue between $[\hat{\mu}, \hat{t}]$ and $[\tilde{\mu}, \tilde{t}]$ does not tell us anything about the difference in total match utility between $\hat{\mu}$ and $\tilde{\mu}$.

## 3.3 Deadweight Loss

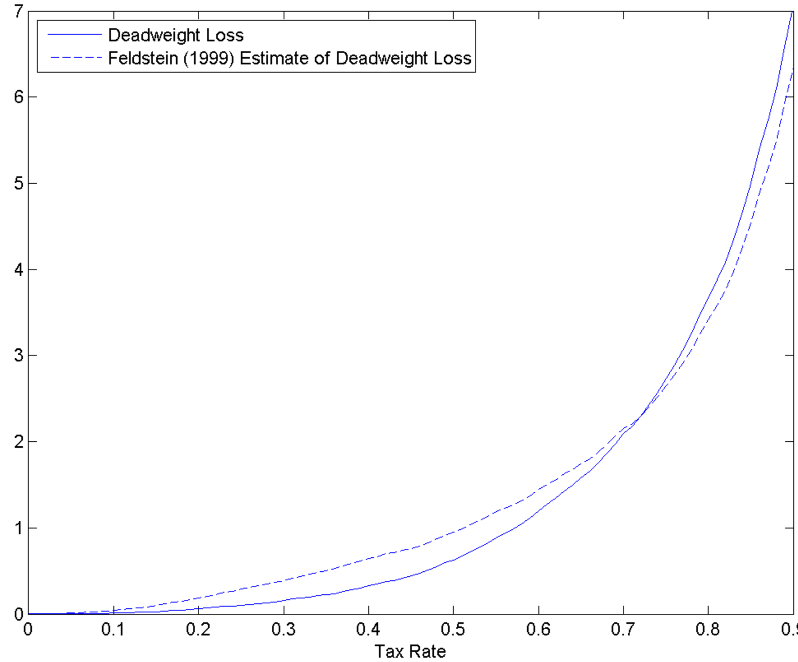
In addition to causing some workers not to work, taxation generates deadweight loss by changing the matching of workers to firms. Thus, workers' decisions on where to work affect managers' productivities and the opportunities available to other workers. These externalities mean that, unlike in the framework of Feldstein (1999), the deadweight loss cannot be calculated from the change in taxable income.[24]

Using the Feldstein (1999) formula,

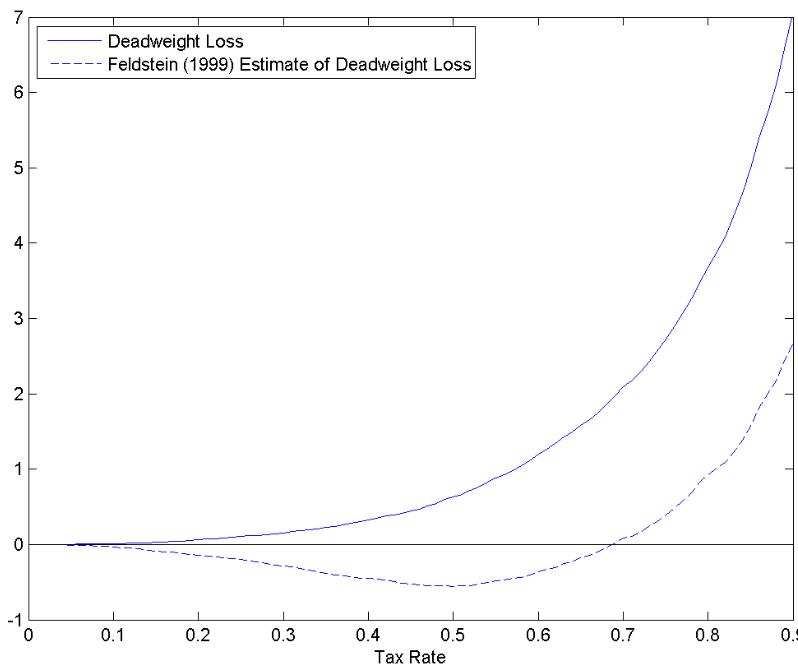
$$\frac{d\text{DWL}}{d\tau} = \tau \frac{d\text{Taxable Income}}{d\tau},$$

can generate substantial bias in our setting. Figure 6 shows the average actual deadweight loss and the average estimated deadweight loss for simulated markets. For 200 markets with 25 agents on each side, we draw match utilities i.i.d. with $\alpha_m^w \sim U[0,1]$ and $\gamma_w^m \sim U[-1,0]$. Figure 6a shows the actual deadweight loss of the worker-optimal stable arrangement and the estimate from the Feldstein (1999) formula; Figure 6b does the same exercise for the manager-optimal stable arrangement. In both cases, the estimated loss can be off by a factor of 2 or more. At the worker-optimal arrangement the Feldstein (1999) estimate is an over-estimate for some tax values and an under-estimate for others. Even more notably, at the manager-optimal stable arrangement, the estimated deadweight loss can actually be negative: At the manager-optimal stable arrangements managers pay workers the bare minimum necessary; when the tax increases, they still need to pay workers a comparable post-tax wage and so the pre-tax wage actually increases.

[24] See, e.g. Chetty (2009) for other conditions under which the Feldstein (1999) formula does not hold.




(a) Deadweight Loss at the Worker-Optimal Stable Arrangementl




(b) Deadweight Loss at the Manager-Optimal Stable Arrangement

Figure 6: Actual and estimated deadweight loss in 200 simulated markets.

Note: The graphs show the average across 200 simulations of one-to-one matching markets with 25 agents on each side of the market. All agents' match utilities are independently and identically distributed with $\alpha_m^w \sim U[0,1]$ and $\gamma_w^m \sim U[-1,0]$. We vary the tax rate, $\tau$, from 0 to .9 in increments of .01. Figure (a) presents deadweight loss for the worker-optimal stable arrangements and Figure (b) refers to the manager-optimal stable arrangements. In each case we calculate the actual deadweight loss at each tax rate and the deadweight loss is estimated based on the formula $\frac{d\text{DWL}}{d\tau} = \tau \frac{d\text{Taxable Income}}{d\tau}$.

# 4 Lump Sum Taxation

While not typically phrased in the exact language of taxation, lump sum taxes are present throughout labor markets. They might take the form of fixed costs per employee (e.g., employee health care costs) or costs of entering employment (e.g., licensing requirements). In the marriage market context, lump sum taxes can take the form of marriage license fees or tax penalties for marriage.

## 4.1 Lump Sum Taxation of Transfers

We first consider a lump sum tax that is levied only on (nonzero) transfers between match partners.[25] Such a *lump sum tax on transfers*, $f$, corresponds to the transfer function

$$\xi_f^{\mathsf{lump}}(t^{m\to w}) \equiv \begin{cases} t^{m\to w} - f & t^{m\to w} \neq 0 \\ t^{m\to w} & t^{m\to w} = 0. \end{cases}$$

Figure 7 shows this transfer function. Under this tax structure, the case $f = 0$ corresponds to the standard (Shapley and Shubik (1971)) model of matching with transfers and the case $f = \infty$ corresponds to (Gale and Shapley (1962)) matching without transfers.

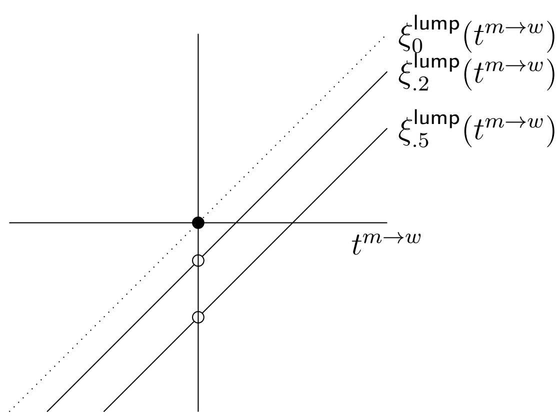



Figure 7: Transfer function $\xi_f^{\mathsf{lump}}(\cdot)$.

We say that an arrangement or matching is *stable under lump sum tax $f$* if it is stable given transfer function $\xi_f^{\mathsf{lump}}(\cdot)$.

A lump sum tax on transfers has an extensive margin effect that makes being unmatched more attractive relative to matching with a transfer. In non-wage markets,[26] a lump sum

[25] An alternative approach to lump sum taxation, which we discuss in the next section, imposes a flat fee on all matches.

[26] Since it is difficult to observe transfers in non-wage markets, such as marriage markets, it is somewhat

tax on transfers can also encourage matchings in which transfers are unnecessary.[27] As our next example illustrates, this second distortion can cause the total match utility of stable matchings to be non-monotonic in the size of the lump sum tax.

Consider a one-to-one market with two managers – $M = \{m_1, m_2\}$ – two workers – $W = \{w_1, w_2\}$ – and match utilities as pictured in Figure 8a. Worker $w_1$ likes $m_1$ – who has a strong preference for $w_2$ – but $w_2$ prefers $m_2$. When transfers are not allowed (or when there is a high lump sum tax on transfers, $f \geq 18$), the only stable matching is the matching $\mu_1$ in which $\mu_1(m_1) = w_1$ and $\mu_1(m_2) = w_2$, as shown in Figure 8b. This matching yields total match utility of $\mathfrak{M}(\mu_1) = 22$.

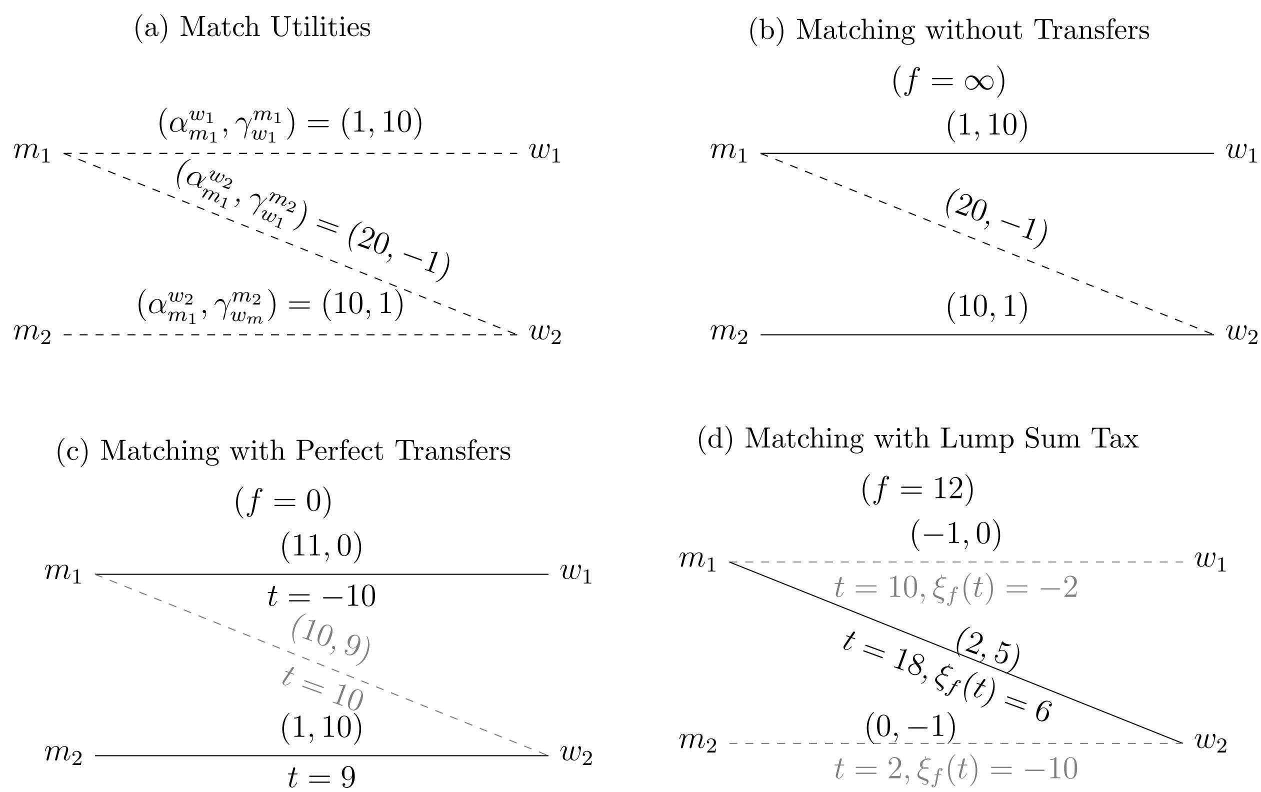



Figure 8: Example 2 – Non-monotonicity under a lump sum tax on transfers.
Note: Utilities, net of transfers, are above the lines (manager's, worker's). Possible supporting transfers (when applicable) are below the lines. Solid lines indicate the stable matching.

**Example 2.** When the lump sum tax is lowered to $f = 12$, only the matching $\mu_2$ is stable, where $\mu_2(m_1) = w_2$ and $w_1$ and $m_2$ are unmatched; this matching gives a total match utility

hard to imagine taxing them. Nevertheless, lump-sum taxes on transfers could correspond to instituting a lump sum tax on gifts between spouses, and flat fees for matching could correspond to requiring marriage license fees.

[27]To see this, consider the case of balanced one-to-one matching markets. In such markets, lump sum taxes on transfers promotes pairing $(m, w)$ in which the match utility $\alpha^w_m + \gamma^m_w$ is evenly distributed between the two partners ($\alpha^w_m \approx \gamma^m_w$), so that transfers are unnecessary.

$\mathfrak{M}(\mu_2) = 19$, as shown in Figure 8d. When $f = 12$, the tax is low enough that $m_1$ can convince $w_2$ to match with him, but not low enough for $w_1$ to hold onto $m_1$ when he has the option of matching with $w_2$ (or $m_2$ to hold onto $w_2$). Lowering the lump sum tax from 20 to 12 decreases the total match utility of the stable matching and decreases the number of agents matched..

Just as in Section 3, we use simulations to confirm that Example 2 is not an exceptional case. We return to the 500 randomly drawn one-to-one markets presented in Section 3, and consider lump sum taxes varying from 0 to 1 in increments of .01. We find that match utility is non-monotonic in the lump sum tax in 61% of our simulated markets.

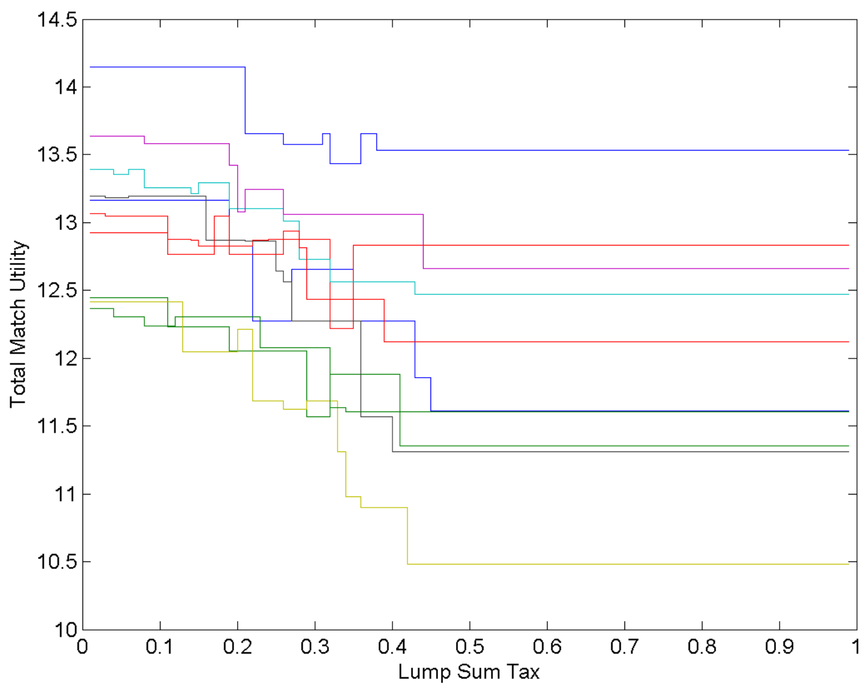



Figure 9: Total match utility of a stable match in ten simulated markets.

Note: The markets presented were randomly-selected from the set of simulated markets exhibiting non-monotonicities. Each market is one-to-one and has 20 agents on each side of the market, with match utilities independently and identically distributed according to a uniform distribution on $[-.5, .5]$. For each tax rate total match utility is calculated for the manager-optimal stable arrangement.

Figure 9 plots the total match utility of the manager-optimal stable match as a function of the tax rate in ten randomly-selected simulation markets with non-monotonicities under lump-sum taxation. In all markets, the total match utility is unchanged for lump sum taxes above .5 because this is the maximum individual match utility. (In equilibrium there are no transfers paid when the tax is .5, so increasing the lump sum tax on transfers above .5 has no effect.)

In strictly positive wage markets, all matchings require a transfer, so a lump sum tax on transfers does not distort agents' preferences among match partners – for a given transfer vector, if a worker prefers manager $m_1$ to $m_2$ without a tax, then that worker also prefers $m_1$ to $m_2$ under a lump sum tax. Thus, in strictly positive wage markets, the matching distortion of the lump sum tax is only on the extensive margin – the decision of *whether* to match – under a higher lump sum tax, fewer agents find matching desirable. This intuition is captured in the following lemma, where we use $\#(\mu)$ to denote the number of workers matched in matching $\mu$

**Lemma 2.** *In strictly positive wage markets, reduction in a lump sum tax on transfers (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, matching $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}).$$

In non-wage markets, the conclusion of Lemma 2 is not true, in general, because distortion among match partners can dominate the extensive margin effect, as in Example 2.

As lump sum taxes do not distort among match partners in strictly positive wage markets, they can only reduce the efficiency of stable matchings in such markets by reducing the number of workers matched. This observation, when combined with Lemma 2, gives the following result.

**Theorem 2.** *In strictly positive wage markets, a reduction in a lump sum tax on transfers (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

Theorem 2 indicates that in strictly positive wage markets, match utility increases monotonically as lump sum taxation decreases.

Just as in the case of proportional taxation, non-monotonicity disappears as a market's structure becomes closer to that of a wage market. Using the same set of simulation markets described in Section 3.2, we analyze how the fraction of markets with non-monotonicities changes as we move from symmetric markets to wage markets. Figure 10 shows the results. We see that a substantial amount of market asymmetry is needed before the fraction of markets with non-monotonicities drops below 50%.

In strictly positive wage markets, we can also bound the total match utility loss from a given lump sum tax.

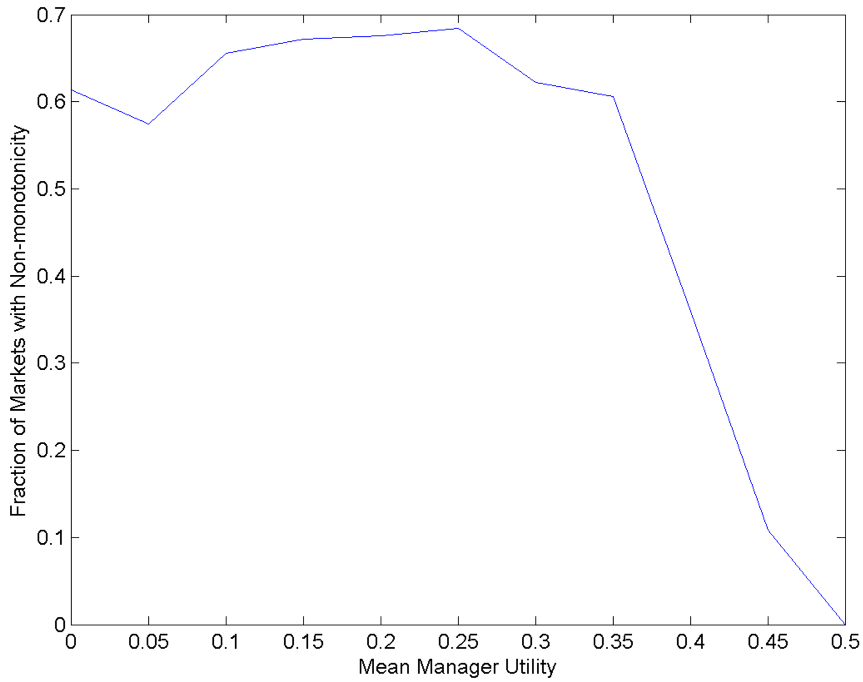



Figure 10: Fraction of simulated markets where the total match utility is non-monotonic in the lump sum tax, $f$.

Note: For each mean manger utility level, we report the fraction of the 500 simulated markets that have a non-monotonicity in total match utility as the lump sum increases from 0 to 1. All simulated markets are one-to-one and have 20 agents on each side of the market.

**Proposition 6.** *In a strictly positive wage market, let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under lump sum tax on transfers $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

The intuition for Proposition 6 is that since the workers unmatched under a lump sum tax of $\tilde{f}$ have negative surplus from matching under that lump sum tax, their surplus from matching could not be more than $\tilde{f}$. So the change in total utility is less than the change in the number of unmatched workers times a maximum surplus of $\tilde{f}$ per worker.

Finally, we can show that, for a *fixed* limit on the number of workers matched in the presence of a lump sum tax, stable matchings in strictly positive wage markets must generate the maximal match utility possible.

**Proposition 7.** *In a strictly positive wage market, a matching $\tilde{\mu}$ can be stable under a lump* *sum tax on transfers only if*

$$\tilde{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} [\mathfrak{M}(\mu)] .$$

Proposition 7 shows that a lump sum tax is an efficient way for a market designer to limit the number of matches (in strictly positive wage markets): the matchings stable under lump sum taxation have maximal utility, given the tax's implied limit on the number of agents matched. Analogously, if a market designer wants to encourage matches, a lump-sum subsidy will maximize total match utility for a given (subsidy-induced) lower bound on the number of agents matched. For example, this suggests that if a government wants to use tuition subsidies to encourage people to go to school, then uniform tuition subsidies are more efficient than subsidies proportional to the cost of tuition.

## 4.2 Lump Sum Taxation of Matches

Some fee structures tax *all* pairings, rather than just those that include nonzero transfers. Such *flat fees for matching* can also be interpreted in the language of taxation: they correspond to the transfer function

$$\xi_f^{\mathsf{fee}}(t^{m \to w}) \equiv t^{m \to w} - f.$$

Figure 11 shows this transfer function for different levels of $f$.

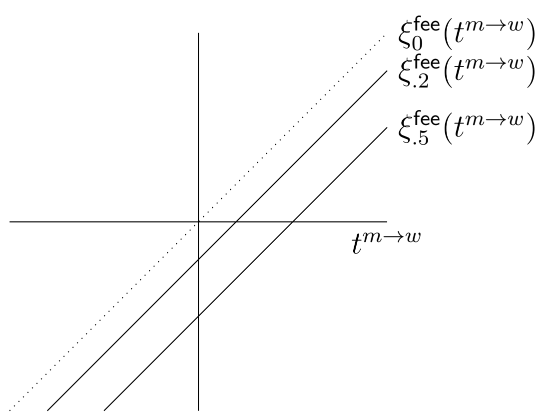



Figure 11: Transfer function $\xi_f^{\mathsf{fee}}(\cdot)$.

Unlike lump sum taxes on transfers, flat fees for matching never distort among match partners – even in non-wage markets. Flat fees for matching only have extensive margin effects, and thus markets with such fees are similar to strictly positive wage markets with lump sum taxes on transfers.[28] As we show in the Appendix, the conclusions of Lemma 2, Theorem 2, and Propositions 6 and 7 *always* hold in markets with flat fees for matching.

# 5 Discussion

Before concluding, we briefly remark on a structural properties common to both models of taxation.

### The Effect of Very Small Taxes

Unlike in non-matching models of taxation, in our setting there is always a non-zero tax that does not generate distortions. To see this in the proportional tax setting, let $\hat{\mu}$ be an efficient matching. Our results show that if $\tilde{\mu}$ is stable under $\tilde{\tau}$, then[29]

$$\tilde{\tau} \geq \frac{\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu})}{\sum_{m \in M}(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)})}. \tag{11}$$

For any inefficient matching $\tilde{\mu}$, there is a strictly positive minimum tax $\underline{\tau}(\tilde{\mu})$ at which $\tilde{\mu}$ could possibly be stable. Since there are finitely many possible matchings, we can just take the minimum of this threshold across inefficient matchings,

$$\underline{\tau}^* = \min_{\{\mu : \mathfrak{M}(\mu) < \mathfrak{M}(\hat{\mu})\}} [\underline{\tau}(\mu)] .$$

For $\tau < \underline{\tau}^*$ only an efficient matching can be stable.[30] The argument for the case of lump sum taxation is similar.

### Structure of the Set of Stable Arrangements

Results of Kelso and Crawford (1982) and Hatfield and Milgrom (2005) imply that for any fixed $\tau$, or $f$, if there are multiple stable arrangements, then workers' and managers' interests are opposed. If all managers prefer $[\mu; t]$ to $[\hat{\mu}; \hat{t}]$, then all workers prefer $[\hat{\mu}; \hat{t}]$ to $[\mu; t]$. Moreover, there exists a manager-optimal (worker-pessimal) stable arrangement that the managers weakly prefer to all other stable arrangements and a worker-optimal (manager-pessimal) stable arrangement that all workers weakly prefer. In wage markets with

[28] Indeed, in strictly positive wage markets, lump sum taxation of transfers is equivalent to lump sum taxation of matchings because workers never match without receiving a strictly positive transfer.

[29] See Equation (41) of the Appendix.

[30] One caveat is that if there are multiple efficient matchings (all of which are stable when $\tau = 0$), some of them may *not* be stable in the limit as $\tau \to 0$ or $f \to 0$.

proportional taxation, where there is generically a unique stable matching, this opposition of interests carries over to the set of supporting transfer vectors.

# 6 Conclusion

We analyze the matching distortion that arises when taxes on transfers affect the matching of workers to managers. In wage markets, matching distortions always decrease as taxes are reduced. In more balanced markets, such as marriage markets or student-college matching, where transfers can flow in either direction, the distortion may be non-monotonic in the amount of taxation or transfer frictions. Non-monotonicities can also occur with piece-wise linear (or curved) taxes because they create changes in the slope of the transfer function, similar to the change in slope that occurs at 0 in non-wage markets.

Matching distortions affect the allocative margin and can arise even without intensive or extensive margin effects. However, our framework allows for extensive margin effects and partially incorporates intensive margin effects, to the extent that changes in work hours are often achieved by changing jobs. An extension of our work would examine a fuller interaction of allocative and intensive margin effects in a model that allows for labor supply decisions within a job. It would also be valuable to analyze how the magnitude of the matching distortion depends on the variance and heterogeneity of agents' preferences. Such work might inform the estimation of the losses due to matching distortions in real-world labor markets.

It is also natural to ask about revenue: How much revenue do different tax structures generate in matching markets? For a given revenue requirement, does a proportional tax generate more or less distortion than a lump sum tax?

The first challenge in answering questions about revenue is that for any stable match, there will generally be a lattice of possible supporting transfer vectors. For proportional taxation, revenue depends on the choice of supporting transfer vector. The easiest transfer vectors to think about are the maximal (worker-optimal) supporting transfer vectors and the minimal (manager-optimal) supporting transfer vectors, which, in wage markets, correspond to maximal and minimal revenue (given the match and tax rate).

Even after focusing on the extremal supporting transfers, addressing revenue questions requires adding structure on agents' match utilities. Unfortunately, it is not clear whether there is a natural structure to impose. Most papers in the matching literature that (unlike our work) do not allow for fully general match utilities assume that the match surplus is a function of one-dimensional agent types. This usually implies that agents agree on the ordinal ranking of agents on the other side of the market. This shifts the distortion to the extensive margin – at any tax level, the most desirable agents on each side will be matched. Moreover, in our framework, just assuming a structure for match surplus is insufficient because the pre-transfer split of match utility, rather than just the total surplus, affects match outcomes in the presence of taxation.

Galichon and Salanié (2014) put enough structure on match utilities to get equations for matchings and surplus without assuming agents agree on the ordinal rankings of match partners. Jaffe is working with Galichon to adapt extensions of the Galichon and Salanié (2014) method to answer questions about deadweight loss and revenue in the presence of taxation. The complexity of this exercise arises not only because of imperfect transfers, but also from the resulting need to separately identify worker and manager match utilities.

Jaffe is also running lab experiments to understand the effects of transfer frictions on matching in a controlled setting. Explicitly dictating match utilities and manipulating tax rates, will allow her to see whether the availability of transfers (and taxes on those transfers) impacts the probability that a market reaches a stable match, and to analyze the transfers agents select.

# Appendix

## Existence of Stable Arrangements

In this section, we use results from the literature on matching with contracts to show the existence of stable arrangements in our framework. For a given transfer vector $t$, the *demand of manager $m \in M$*, denoted $D^m(t)$, is

$$D^m(t) \equiv \arg\max_{Y \subseteq W} \left\{ \alpha_m^Y - t^{m \to Y} \right\}.$$

**Definition** (Kelso and Crawford (1982))**.** The preferences of manager $m \in M$ are *substitutable* if for any transfer vectors $t$ and $\tilde{t}$ with $\tilde{t} \geq t$, there exists, for each $Y \in D^m(t)$, some $\tilde{Y} \in D^m(\tilde{t})$ such that

$$\tilde{Y} \supseteq \{w \in Y : t^{m \to w} = \tilde{t}^{m \to w}\}.$$

That is, the preferences of $m \in M$ are substitutable if an increase in the "prices" of some workers cannot decrease demand for the workers whose prices remain unchanged.[31]

Theorem 2 of Kelso and Crawford (1982) shows that under the assumption that all managers' preferences are substitutable, there is an arrangement $[\mu; t]$ that is *strict core*, in the sense that:[32]

- Each agent (weakly) prefers his assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad \forall i \in M \cup W.$$

- There does not exist a manager $m \in M$, a set of workers $Y \subseteq W$, and a transfer vector $\tilde{t}$ such that

$$\begin{aligned} \alpha_m^Y - \tilde{t}^{m \to Y} &\geq \alpha_m^{\mu(m)} - t^{m \to \mu(m)}, && \text{and} \\ \gamma_w^m + \xi(\tilde{t}^{m \to w}) &\geq \gamma_w^{\mu(w)} + \xi(t^{\mu(w) \to w}) && \forall w \in Y, \end{aligned}$$

  with strict inequality for at least one $i \in (\{m\} \cup Y)$.

The Kelso and Crawford (1982) (p. 1487) construction of competitive equilibria from strict core allocations then implies that there is some transfer vector $\hat{t}$, having $\hat{t}^{\mu(w) \to w} = t^{\mu(w) \to w}$ (for each $w \in W$), such that $[\mu; \hat{t}]$ is stable in our sense.

[31] Theorem A.1 of Hatfield et al. (2013) shows that in our setting the Kelso and Crawford (1982) substitutability condition is equivalent to the choice-based substitutability condition of Hatfield and Milgrom (2005), that we describe in the main text: *the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.*

[32] Strictly speaking, Kelso and Crawford (1982) have one technical assumption not present in our framework: they assume that $\alpha_m^w + \gamma_w^m \geq 0$, in order to ensure that all workers are matched. However, examining the Kelso and Crawford (1982) arguments reveals that this extra assumption is not necessary to ensure that a strict core arrangement exists – the Kelso and Crawford (1982) salary adjustment processes can be started at some arbitrarily low (negative) salary offer and all of the steps and results of Kelso and Crawford (1982) remain valid, with the caveat that some workers may be unmatched at core outcomes.

## Proof of Lemma 1

We let $\mathfrak{B}$ be the set of managers who are matched at $\mu$ and let $\mathscr{B}$ be the set of workers who are matched at $\mu$. This means that

$$\begin{aligned} &\mu(m) \subseteq \mathscr{B} && \forall m \in \mathfrak{B}, \\ &\mu(w) \in \mathfrak{B} && \forall w \in \mathscr{B}. \end{aligned}$$

These observations, combined with the fact that $t^{m\to m} = t^{w\to w} = 0$, enable us to show that

$$\begin{aligned} \sum_{m\in M} t^{m\to\mu(m)} &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)} + \sum_{m\in M\setminus\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} \sum_{w\in\mu(m)} t^{m\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w} + \sum_{w\in W\setminus\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in W} t^{\mu(w)\to w}. \end{aligned}$$

## Proof of Proposition 1

First, we show that the arrangements stable under full taxation ($\tilde{\tau} = 1$) cannot Pareto dominate those stable under tax $\hat{\tau} < 1$.

**Claim.** *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau} < 1$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau} = 1$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau} < 1$).*

*Proof.* As no transfers get through under full taxation, an arrangement stable under full taxation is most likely to Pareto dominate some other arrangement when all transfers between match partners are 0. Thus, we assume that $\tilde{t}^{\tilde{\mu}(w)\to w} = 0$ for each $w \in W$, and suppose that $[\tilde{\mu}; \tilde{t}]$ (under full taxation) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$). This would imply that

$$\alpha_m^{\tilde{\mu}(m)} = \alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)}, \tag{12}$$
$$\gamma_w^{\tilde{\mu}(w)} = \gamma_w^{\tilde{\mu}(w)} + \xi_{\tilde{\tau}}^{\mathsf{prop}}(\tilde{t}^{\tilde{\mu}(w)\to w}) \geq \gamma_w^{\hat{\mu}(w)} + \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\hat{\mu}(w)\to w}), \tag{13}$$

with strict inequality for some $m$ or $w$. However, stability of $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ implies that

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{14}$$
$$\gamma_w^{\hat{\mu}(w)} + \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\hat{\mu}(w)\to w}) \geq \gamma_w^{\tilde{\mu}(w)} + \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w)\to w}). \tag{15}$$

Combining (12) and (14) gives

$$0 \geq -\hat{t}^{m \to \tilde{\mu}(m)}, \tag{16}$$

for each $m \in M$, while combining (13) and (15) gives

$$0 \geq \xi_{\tilde{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w) \to w}), \tag{17}$$

for each $w \in W$. Strict inequality must hold in (16) or (17) for some $m$ or $w$.

In the first of these cases, we have

$$\hat{t}^{m' \to \tilde{\mu}(m')} > 0$$

for some $m' \in M$; hence, there exists at least one $w \in \hat{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w) \to w} > 0. \tag{18}$$

But (18) contradicts (17).

In the second case, we have

$$0 > \xi_{\tilde{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w') \to w'}), \tag{19}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (19) and (17) together imply that

$$0 > \sum_{w \in \tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w) \to w} = \hat{t}^{m \to \tilde{\mu}(m)},$$

contradicting (16). □

For $\tilde{\tau} < 1$, $\xi_{\tilde{\tau}}^{\mathsf{prop}}(\cdot)$ is strictly increasing and the conclusion of the proposition follows from the following more general result.

**Proposition 1'.** *Suppose that $\tilde{\xi}(\cdot)$ is strictly increasing, that $[\hat{\mu}; \hat{t}]$ is stable under $\hat{\xi}(\cdot)$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under $\tilde{\xi}(\cdot)$, with $\tilde{\xi}(\cdot) \leq \hat{\xi}(\cdot)$. Then, $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$).*[33]

*Proof.* The case for $\tilde{\tau}$ Pareto dominance of $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) over $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$) would imply that

$$\alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)}, \tag{20}$$
$$\gamma_w^{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma_w^{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}), \tag{21}$$

[33]We say that an arrangement $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) *Pareto dominates* arrangement $[\hat{\mu}; \hat{t}]$ under (under $\hat{\xi}(\cdot)$) if

$$\alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \qquad \forall m \in M,$$
$$\gamma_w^{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma_w^{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}) \qquad \forall w \in W,$$

with strict inequality for some $i \in M \cup W$.

with strict inequality for some $m$ or $w$. However, stability of $[\hat{\mu};\hat{t}]$ under $\hat{\xi}(\cdot)$ implies that

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{22}$$

$$\gamma_w^{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w)\to w}) \geq \gamma_w^{\tilde{\mu}(w)} + \hat{\xi}(\hat{t}^{\tilde{\mu}(w)\to w}) \geq \gamma_w^{\tilde{\mu}(w)} + \tilde{\xi}(\hat{t}^{\tilde{\mu}(w)\to w}), \tag{23}$$

where the second inequality in (23) follows from the fact that $\hat{\xi}(\cdot) \geq \tilde{\xi}(\cdot)$.

Combining (20) and (22) gives

$$\hat{t}^{m\to\tilde{\mu}(m)} \geq \tilde{t}^{m\to\tilde{\mu}(m)}, \tag{24}$$

for each $m \in M$, while combining (21) and (23) gives

$$\begin{aligned} \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w)\to w}) &\geq \tilde{\xi}(\hat{t}^{\tilde{\mu}(w)\to w}) \\ \tilde{t}^{\tilde{\mu}(w)\to w} &\geq \hat{t}^{\tilde{\mu}(w)\to w} \end{aligned} \tag{25}$$

for each $w \in W$, where the second line of (25) follows from the fact that $\tilde{\xi}(\cdot)$ is strictly increasing. Strict inequality must hold in (24) or (25) for some $m$ or $w$.

In the first of these cases, we have

$$\hat{t}^{m'\to\tilde{\mu}(m')} > \tilde{t}^{m'\to\tilde{\mu}(m')}$$

for some $m' \in M$; hence, there exists at least one $w \in \tilde{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w)\to w} > \tilde{t}^{\tilde{\mu}(w)\to w}. \tag{26}$$

But (26) contradicts (25).

In the second case, we have

$$\tilde{t}^{\tilde{\mu}(w')\to w'} > \hat{t}^{\tilde{\mu}(w')\to w'} \tag{27}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (27) and (25) together imply that

$$\sum_{w\in\tilde{\mu}(m)} \tilde{t}^{\tilde{\mu}(w)\to w} > \sum_{w\in\tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w)\to w};$$

hence, we find that

$$\tilde{t}^{m\to\tilde{\mu}(m)} > \hat{t}^{m\to\tilde{\mu}(m)},$$

contradicting (24). □

## Proof of Theorem 1

If $\hat{\mu} = \tilde{\mu}$, then the theorem is trivially true. Thus, we consider a wage market in which $[\tilde{\mu};\tilde{t}]$ is stable under tax $\tilde{\tau}$, $[\hat{\mu};\hat{t}]$ is stable under tax $\hat{\tau}$, $\tilde{\tau} > \hat{\tau}$, and $\tilde{\mu} \neq \hat{\mu}$.

The stability conditions for the managers imply that

$$\alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \tilde{t}^{m\to\hat{\mu}(m)}, \tag{28}$$
$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}; \tag{29}$$

these inequalities together imply that

$$\sum_{m\in M} \left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right) \geq \sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \tag{30}$$

As the market is a wage market, we have

$$\xi_\tau^{\mathsf{prop}}(\tilde{t}^{\tilde{\mu}(w)\to w}) = (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \quad \text{and} \quad \xi_\tau^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w)\to w}) = (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w};$$

hence, the stability conditions for the workers imply that

$$\gamma_w^{\tilde{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma_w^{\hat{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\hat{\mu}(w)\to w}, \tag{31}$$
$$\gamma_w^{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \gamma_w^{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w}. \tag{32}$$

Summing these inequalities and applying Lemma 1, we obtain

$$(1-\hat{\tau})\sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M} \left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right). \tag{33}$$

Combining (30) and (33), we find that

$$(1-\hat{\tau})\sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \tag{34}$$

Since $\hat{\tau} < \tilde{\tau}$, (34) implies that

$$\sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0. \tag{35}$$

Next, using (29) and (32), we find that

$$\begin{aligned}
\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m\in M} (\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}) + \sum_{w\in W} (\gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)}) \\
&\geq \sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) - (1-\hat{\tau})\sum_{w\in W} \left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right), \\
&= \hat{\tau}\sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0,
\end{aligned}$$

where the final inequality follows from (35).

## Proof of Proposition 2 and Derivation of Equation (11)

Summing (31) across women, we find that

$$\sum_{w\in W}\left(\gamma_w^{\tilde{\mu}(w)}-\gamma_w^{\hat{\mu}(w)}\right) \geq (1-\tilde{\tau})\sum_{w\in W}\left(\tilde{t}^{\hat{\mu}(w)\to w}-\tilde{t}^{\tilde{\mu}(w)\to w}\right) \tag{36}$$

$$\geq (1-\tilde{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w}-\hat{t}^{\tilde{\mu}(w)\to w}\right) \tag{37}$$

$$\geq 0, \tag{38}$$

where the inequality (37) follows from (30), and the inequality (38) follows from (35). Thus, we see Proposition 2 – the workers receive higher match utility under $\tilde{\mu}$ than under $\hat{\mu}$.

Furthermore, this implies that

$$\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right) \geq 0, \tag{39}$$

so that we may calculate the lowest tax under which a given inefficient match $\tilde{\mu}$ can be stable. Combining (28) and (36), we find that

$$\sum_{w\in W}\left(\gamma_w^{\tilde{\mu}(w)}-\gamma_w^{\hat{\mu}(w)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right). \tag{40}$$

The inequality in (39) allows us to rearrange (40) to obtain

$$\frac{\sum_{w\in W}\left(\gamma_w^{\tilde{\mu}(w)}-\gamma_w^{\hat{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right)} \geq (1-\tilde{\tau}),$$

so that we find

$$\tilde{\tau} \geq \frac{\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right)}{\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right)}+\frac{\sum_{w\in W}\left(\gamma_w^{\hat{\mu}(w)}-\gamma_w^{\tilde{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right)}$$

$$=\frac{\mathfrak{M}(\hat{\mu})-\mathfrak{M}(\tilde{\mu})}{\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)}-\alpha_m^{\tilde{\mu}(m)}\right)}. \tag{41}$$

## Proof of Proposition 3

Assume a matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$. In a wage market, if we re-normalize the workers utilities by dividing by $(1-\tilde{\tau})$, then a match that is stable with the renormalized utilities and no taxation is also stable with the original utilities and tax $\tilde{\tau}$

$$\gamma_w^{\tilde{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma_w^{m)} + (1-\tilde{\tau})\tilde{t}^{m\to w},$$
$$\iff \frac{1}{1-\tilde{\tau}}\gamma_w^{\tilde{\mu}(w)} + \tilde{t}^{\hat{\mu}(w)\to w} \geq \frac{1}{1-\tilde{\tau}}\gamma_w^{m} + \tilde{t}^{m\to w}. \tag{42}$$

Combining (42) with the standard manager stability conditions,

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^Y - \hat{t}^{m\to Y},$$

gives a matching market with quasilinear utility. It is known (e.g., Kelso and Crawford (1982); Hatfield et al. (2013)) that in such markets, only an efficient matching can stable. So $\tilde{\mu}$ must maximize the total of the re-normalized match utilities,

$$\tilde{\mu} \in \arg\max_{\{\mu\}} \left[\sum_{m\in M} \alpha_m^{\mu(m)} + \sum_{w\in W} \frac{1}{(1-\tilde{\tau})}\gamma_w^{\mu(w)}\right].$$

## Proofs of Proposition 4 and Corollary 1

Suppose that in a wage market, both $[\tilde{\mu};\tilde{t}]$ and $[\hat{\mu};\hat{t}]$ are stable under tax $\tau$. The stability conditions for the managers imply that

$$\alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \tilde{t}^{m\to\hat{\mu}(m)}, \tag{43}$$
$$\alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)} \leq \alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)}, \tag{44}$$

so that

$$\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}. \tag{45}$$

Meanwhile, the stability conditions for the workers imply that

$$\gamma_w^{\tilde{\mu}(w)} + (1-\tau)\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma_w^{\hat{\mu}(w)} + (1-\tau)\tilde{t}^{\hat{\mu}(w)\to w}, \tag{46}$$
$$\gamma_w^{\tilde{\mu}(w)} + (1-\tau)\hat{t}^{\tilde{\mu}(w)\to w} \leq \gamma_m^{\hat{\mu}(w)} + (1-\tau)\hat{t}^{\hat{\mu}(w)\to w}, \tag{47}$$

so that

$$(1-\tau)(\tilde{t}^{\hat{\mu}(w)\to w} - \tilde{t}^{\tilde{\mu}(w)\to w}) \leq (1-\tau)(\hat{t}^{\hat{\mu}(m)\to w} - \hat{t}^{\tilde{\mu}(m)\to w}). \tag{48}$$

Summing (45) and (48) across agents and using Lemma 1, we find that

$$\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right) = \sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right).$$

For this to hold, we must have equality in (45) for each $m \in M$. But this implies equality in (43) and (44), for each $m \in M$. Similarly, it requires that (48) hold with equality for each $w \in W$, which implies equality in (46) and (47), for each $w \in W$. Combining these equalities, and summing across workers $w \in W$, shows that

$$\sum_{w\in W}\left(\gamma_w^{\tilde{\mu}(w)} - \gamma_w^{\hat{\mu}(w)}\right) = (1-\tau)\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right), \\ = (1-\tau)\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}\right). \tag{49}$$

If the managers are not indifferent in aggregate between $\tilde{\mu}$ and $\hat{\mu}$, so that

$$\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}\right) \neq 0, \tag{50}$$

we have,

$$\tau = 1 + \frac{\sum_{w\in W}\left(\gamma_w^{\tilde{\mu}(w)} - \gamma_w^{\hat{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha_m^{\tilde{\mu}(m)} - \alpha_m^{\hat{\mu}(m)}\right)}. \tag{51}$$

This shows Proposition 4.

To see Corollary 1, it suffices to observe that (51) pins down a unique tax rate in the case that (50) holds. Thus, if there are two tax rates under which matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable, then we must have

$$\sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}\right) = 0. \tag{52}$$

But then, we also have

$$\sum_{w\in W}\left(\gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)}\right) = 0, \tag{53}$$

by (49). Combining (52) and (53), we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{m\in M}\left(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}\right) + \sum_{w\in W}\left(\gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)}\right) = 0,$$

as desired.

## Proof of Proposition 5

Follows directly from the arguments presented the text.

## Proof of Lemma 2

In a strictly positive wage market, all matches are accompanied by a strictly positive transfer; hence, a lump sum tax on transfers is equivalent to a flat fee for matching. Thus, Lemma 2 follows from the following slightly more general result.

Here and hereafter, we say that an arrangement or matching is *stable under flat fee $f$* if it is stable given transfer function $\xi_f^{\mathsf{fee}}(\cdot)$.

**Lemma 2'.** *Reduction of a flat fee for matching (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, matching $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}),$$

*where $\#(\mu)$ denotes the number of workers matched in matching $\mu$.*

*Proof.* As $[\tilde{\mu}; \tilde{t}]$ is stable under flat fee $\tilde{f}$, we have

$$\alpha_m^{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha_m^{\hat{\mu}(m)} - \tilde{t}^{m \to \hat{\mu}(m)}$$
$$\gamma_w^{\tilde{\mu}(w)} + \tilde{t}^{\tilde{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\} \geq \gamma_w^{\hat{\mu}(w)} + \tilde{t}^{\hat{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\hat{\mu}(w) \neq w}\}$$

where $\{1_{\mu(w) \neq w}\}$ is an indicator function that equals 1 if $w$ is matched in matching $\mu$ and 0 if $w$ is unmatched in matching $\mu$. Summing these inequalities across agents, and using Lemma 1, we find that

$$\sum_{m \in M} \left( \alpha_m^{\tilde{\mu}(m)} - \alpha_m^{\hat{\mu}(m)} \right) + \sum_{w \in W} \left( \gamma_w^{\tilde{\mu}(w)} - \gamma_w^{\hat{\mu}(w)} \right) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{54}$$

Similarly, as $[\hat{\mu}; \hat{t}]$ is stable under flat fee $\hat{f}$,

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}$$
$$\gamma_w^{\hat{\mu}(w)} + \hat{t}^{\hat{\mu}(w) \to w} - \hat{f} \cdot \{1_{\hat{\mu}(w) \neq w}\} \geq \gamma_w^{\tilde{\mu}(w)} + \hat{t}^{\tilde{\mu}(w) \to w} - \hat{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\};$$

these inequalities yield

$$\sum_{m \in M} \left( \alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)} \right) + \sum_{w \in W} \left( \gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)} \right) + \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0. \tag{55}$$

upon summation.

Adding (54) and (55) shows that

$$(\tilde{f} - \hat{f})(\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0.$$

Thus, if $\tilde{f} > \hat{f}$, we must have $\#(\hat{\mu}) \geq \#(\tilde{\mu})$; this proves the result. □

# Proof of Theorem 2

As in the proof of Lemma 2, Theorem 2 follows from the following slightly more general result.

**Theorem 2'.** *A reduction in a flat fee for matching (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

*Proof.* Using (55) and Lemma 2', we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{m \in M} \left(\alpha_m^{\hat{\mu}(m)} - \alpha_m^{\tilde{\mu}(m)}\right) + \sum_{w \in W} \left(\gamma_w^{\hat{\mu}(w)} - \gamma_w^{\tilde{\mu}(w)}\right) \geq \hat{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0; \tag{56}$$

this proves Theorem 2'. ☐

## Proof of Proposition 6

As in the proof of Lemma 2, Proposition 6 follows from the following slightly more general result.

**Proposition 6'.** *Let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under flat fee $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

*Proof.* This is immediate from (54). ☐

## Proof of Proposition 7

As in the proof of Lemma 2, Proposition 7 follows from the following slightly more general result.

**Proposition 7'.** *A matching $\tilde{\mu}$ can be stable under a flat fee only if*

$$\tilde{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} \left\{\mathfrak{M}(\mu)\right\}.$$

*Proof.* From (54), we see that if $[\tilde{\mu}; \tilde{t}]$ is stable under flat fee $\tilde{f}$, then for any matching $\hat{\mu} \neq \tilde{\mu}$,

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{57}$$

If fewer workers are matched in $\hat{\mu}$ than in $\tilde{\mu}$ (i.e. $\#(\tilde{\mu}) \geq \#(\hat{\mu})$), (57) implies that

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) \geq \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0,$$

so that $\tilde{\mu}$ must have higher total match utility than $\hat{\mu}$. ☐

# References

**Arcidiacono, Peter, Andrew Beauchamp, and Marjorie McElroy**, "Terms of endearment: An equilibrium model of sex and matching," 2011. NBER Working Paper No. 16517.

**Becker, Gary S.**, "A theory of marriage: Part II," *Journal of Political Economy*, 1974, *82*, 11–26.

**Blundell, Richard, Alan Duncan, and Costas Meghir**, "Estimating labor supply responses using tax reforms," *Econometrica*, 1998, *66*, 827–861.

**Boadway, R., M. Marchand, and P. Pestieau**, "Optimal linear income taxation in models with occupational choice," *Journal of Public Economics*, 1991, *46* (2), 133–162.

**Boone, Jan and Lans Bovenberg**, "Optimal labour taxation and search," *Journal of Public Economics*, 2002, *85* (1), 53–97.

**Chetty, Raj**, "Is the taxable income elasticity sufficient to calculate deadweight loss? the implications of evasion and avoidance," *American Economic Journal: Economic Policy*, 2009, pp. 31–52.

**Crawford, Vincent P. and Elsie Marie Knoer**, "Job matching with heterogeneous firms and workers," *Econometrica*, 1981, *49*, 437–450.

**Feldstein, Martin**, "Tax avoidance and the deadweight loss of the income tax," *The Review of Economics and Statistics*, 1999, *81* (4), 674–680.

**Gale, David and Lloyd S. Shapley**, "College admissions and the stability of marriage," *American Mathematical Monthly*, 1962, *69*, 9–15.

**Galichon, Alfred and Bernard Salanié**, "Cupid's invisible hand: Social surplus and identification in matching models," 2014. Mimeo.

**Gentry, William M. and R. Glenn Hubbard**, "The effects of progressive income taxation on job turnover," *Journal of Public Economics*, 2004, *88* (11), 2301–2322.

**Hatfield, John William and Paul Milgrom**, "Matching with contracts," *American Economic Review*, 2005, *95*, 913–935.

**_ , Scott Duke Kominers, Alexandru Nichifor, Michael Ostrovsky, and Alexander Westkamp**, "Stability and competitive equilibrium in trading networks," *Journal of Political Economy*, 2013, *121*, 966–1005.

**Holzner, Christian and Andrey Launov**, "Do employed workers search efficiently? implications for the income tax system," 2012. Mimeo.

**Kelso, Alexander S. and Vincent P. Crawford**, "Job matching, coalition formation, and gross substitutes," *Econometrica*, 1982, *50*, 1483–1504.

**Koopmans, Tjalling C. and Martin Beckmann**, "Assignment problems and the location of economic activities," *Econometrica*, 1957, *25* (1), 53–76.

**Legros, Patrick and Andrew F. Newman**, "Beauty is a beast, frog is a prince: Assortative matching with nontransferabilities," *Econometrica*, 2007, *75*, 1073–1102.

**Lockwood, Benjamin, Charles Nathanson, and E. Glen Weyl**, "Taxation and the allocation of talent," 2013. Mimeo.

**Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green**, *Microeconomic Theory*, Oxford University Press, 1995.

**Meyer, Bruce D.**, "Labor supply at the extensive and intensive margins: The EITC, welfare, and hours worked," *American Economic Review*, 2002, *92* (2), 373–379.

**Mortensen, Dale and Christopher Pissarides**, "Taxes, subsidies and equilibrium labour market outcomes," 2001. Mimeo.

**Parker, Simon C.**, "Does tax evasion affect occupational choice?," *Oxford Bulletin of Economics and Statistics*, 2003, *65* (3), 379–394.

**Powell, David and Hui Shan**, "Income taxes, compensating differentials, and occupational choice: How taxes distort the wage-amenity decision," *American Economic Journal: Economic Policy*, 2012, *4* (1), 224–247.

**Quinzii, Martine**, "Core and competitive equilibria with indivisibilities," *International Journal of Game Theory*, 1984, *135*, 41–60.

**Roth, Alvin E.**, "The economics of matching: Stability and incentives," *Mathematics of Operations Research*, 1982, *7*, 617–628.

**Rothschild, Casey and Florian Scheuer**, "Redistributive taxation in the roy model*," *The Quarterly Journal of Economics*, 2012, p. qjs076.

**Saez, Emmanuel**, "Optimal income transfer programs: intensive versus extensive labor supply responses," *Quarterly Journal of Economics*, 2002, *117* (3), 1039–1073.

_ , "Reported incomes and marginal tax rates, 1960-2000: evidence and policy implications," in James Poterba, ed., *Tax Policy and the Economy*, MIT Press, 2004.

**Shapley, Lloyd S. and Martin Shubik**, "The assignment game I: The core," *International Journal of Game Theory*, 1971, *1*, 111–130.

**Sheshinski, Eytan**, "Note on income taxation and occupational choice," 2003. CESifo Working Paper No. 880.