Concordance among Holdouts*

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Abstract

Holdout problems prevent decentralized aggregation of complementary goods, but the coercion required to overcome holdout may encourage abuse and violate fairness standards. We propose second-best efficiency, abuse-prevention, and fairness criteria for procedures intended to reduce holdout. Our criteria are jointly satisfied by a class of "Concordance" procedures. In these procedures, the prospective buyer makes a take-it-or-leave-it offer to the group of sellers, and the sellers use an efficient collective choice mechanism to decide as a group whether to accept the buyer's offer. In the case of sale, the buyer's offer is divided among the sellers in a fashion independent of individual sellers' actions. Each seller retains the option to receive, in the event of sale, her share of the offer (without making any additional payments) in exchange for not influencing the collective decision. Our approach is applicable in a range of contexts including land assembly, spectrum aggregation, corporate acquisitions, and patent pool formation.

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I Introduction

Developers assembling land, acquirers of large corporations, founders of patent pools and wireless internet providers in search of spectrum all must obtain the consent of many self-interested sellers of complementary components.[1] Component prices rise as sellers seek shares of the surplus created by the components' assembly, often rendering buyers' intended projects uneconomic.[2] To avoid such seller "holdout," governments commonly employ coercive, expropriatory solutions (such as the power of "eminent domain") to facilitate assembly.

Unfortunately, coercive assembly undermines property rights, inviting abusive takings by buyers and unfair redistribution of property. In this paper, we propose a series of design criteria that strike a balance between efficiency, abuse-prevention, and fairness, along with a class of "Concordance" procedures that achieve these goals.

As we explain in Sections II and III, the holdout problem is three-fold. First, because assembly is necessarily a public bad, affecting all sellers or none, the Mailath and Postelwaite (1990) Theorem implies that no private (voluntary and self-financing) transaction procedure achieves assembly with positive probability as the number of sellers grows large. This necessitates the use of at least some coercion. However, unlike public goods, assemblies are typically proposed by (potentially) opportunistic buyers with private information. Unless sellers can provide some check on coercive assembly, these buyers will abuse coercion to acquire the most valuable plots. This leads naturally to a desire to provide sellers with some ability to veto exploitative purchases. Unfortunately, if such a veto is provided through standard mechanisms, such as voting or the common implementation of the Vickrey (1961)-Clarke (1971)-Groves (1973) mechanism, resources are typically unfairly redistributed—both among and away from sellers. Rather than seeking an optimal trade-off among these disparate and not easily commensurable challenges, we adopt a market design approach. In particular, we formalize and advocate second-best properties along each of these dimensions, and then show that those properties are satisfied by a natural class of transaction procedures.

Section IV presents our proposed design criteria. First, we argue that inefficiency should be limited to that implied by the existence of an underlying bilateral bargain between the buyer and the community of sellers. Second, we argue that sellers should have enough veto power to prevent inefficient assemblies. Finally, we argue for procedures that both

1. ensure that by abstaining from influencing the sale decision sellers can guarantee that

[1] As we discuss below, our other work (Kominers and Weyl, 2012) explored the extent to which holdout problems arise even when perfect complementarity fails.

[2] This problem is ubiquitous in economics. First formalized by Cournot (1838), it takes its precise modern form in the work of Mailath and Postelwaite (1990). It is often known as "double marginalization" (Spengler, 1950) or "anticommons" (Michelman, 1967).

they receive a fair share of the buyer's offer in case of sale and pay nothing if sale does not occur, and

2. incentivize purchasers to divide their offers according to sellers' true (expected) shares of the total value.

These two properties, combined with the absence of inefficient assemblies, ensure that each seller's participation is "approximately individually rational," in that she retains the option to receive, conditional on a sale occurring, an unbiased estimate of the value of her property.

In Section V, we propose a class of *Concordance procedures*, inspired by Cournot's theory of collaboration (*concours*) among producers, which implement our key design goals. These procedures treat the entire group of sellers as a "community," which receives an offer from the buyer and decides whether to accept it through an efficient collective choice mechanism. Sellers divide the proceeds of sale according to shares specified by the buyer, and each seller retains the option to receive her share of the offer (without making any additional payments) in exchange for not influencing the collective decision.

As we discuss in Section VI, the effectiveness of Concordance procedures depends crucially on the efficiency of the collective decision making mechanism used. Simple procedures have limitations and tradeoffs familiar from auction theory. A new approach, using the "Quadratic Vote Buying" mechanism of Weyl (2012), may be particularly promising.

For concreteness we develop most our analysis in the context of land assembly, but the ideas we present have broader applications. In previous work (Kominers and Weyl, 2012), we focused on the example of spectrum reassembly. In Section VII, we illustrate how our framework applies to two other important holdout settings: corporate acquisitions and patent pool formation.[3]

Relation to the literature

The previous literature on holdout has been divided into two parts. The first has documented the inefficiencies created by holdout, both theoretically (Menezes and Pitchford, 2004; Miceli

[3]Back-of-the-envelope calculations suggest that the annual volume of these transactions subject to holdout is on the order of trillions of dollars per year: In the United States alone, there are nearly 6000 active takings per year (Berliner, 2006); these likely represent only a small fraction of all land assembly in the United States. Supposing an average stake of \$10 million, these represent \$60 billion annually; similar guesses for México and Brazil indicate together at least \$20 billion dollars annually. Thus global land assembly activity is likely on the order of hundreds of billions of dollars each year. According to Dealogic, corporate acquisitions amounted to \$972 billion or 5.5% of global GDP in the first quarter of 2008 alone (Twaronite, 2009). When the economy is weak, reduced acquisitions are compensated by debt settlements; in 2008, according to BankruptcyData.com, the assets of United States firms filing for bankruptcy amounted to more than a trillion dollars. Aggregating these examples gives our multi-trillion dollar estimate.

and Segerson, 2007) and empirically (Sorensen, 1999). The second has proposed procedures that (like eminent domain) achieve efficiency by sacrificing the other design goals entirely. We discuss one such example—the mechanism of Plassmann and Tideman (2009), which is not individually rational—in Section III.C.[4]

Formally, land assembly problems are equivalent, conditional on buyer behavior, to (binary) public goods problems[5]; thus, our work fits within the broader literature on public goods. A number of papers have identified environments in which public goods provision problems do not arise, either because information is close to complete (Groves and Ledyard, 1977; Hurwicz, 1977; Walker, 1981; Tian, 1989; Bagnoli and Lipman, 1988) or because information becomes complete as the number of sellers grows (Hellwig, 2003).[6] Other papers have explored the role that altruistic preferences can play in enabling public good provision (Andreoni, 2007). While these works have identified environments in which holdout-reducing procedures are unnecessary, the fact that they leave many holdout settings uncovered is likely an important part of the motivation for the use of coercion in practice. Some work on public goods mechanism design has been able to achieve efficiency in general, but in doing so has sacrificed individual rationality and fairness (Clarke, 1971).

Thus, our approach differs from the previous literature in two ways: First, we focus on how mechanisms for reducing holdout can encourage buyer opportunism; this possibility does not arise in standard public goods settings and has not been addressed in prior analysis of holdout. Second, we seek to balance competing design goals—enabling efficient assembly, preventing buyer opportunism, and ensuring fairness—rather than restricting to environments where these goals are not in conflict or focusing only on a single goal. We consider balance important in general and essential for applications, such as those discussed in Section VII.

[4]Other authors have proposed individually rational land assembly procedures that eliminate strategic misrepresentation of valuations (Grossman et al., 2010). Unfortunately, the logic of Mailath and Postelwaite (1990) implies that these mechanisms do not solve the underlying holdout problem: they lead to no trade with probability 1 in large markets (Shavell, 2010).

[5]"Seller values" become willingnesses to pay and the "buyer offer" becomes the (exogenous) cost of the project. True shares are Lindahl prices and actual (approximate) shares are closely connected to the *pseudo-Lindahl* prices of Bergstrom (1979), a public authority's closest approximation to Lindahl based on public information. Thus Concordance's guarantee of approximate individual rationality implies a corresponding binary public goods mechanism that implements the (efficient) *pseudo-Lindahl* equilibrium. As far as we know, our work is the first that does this in general.

A simple example is the reverse of land assembly—land reform. All of our procedures apply just as easily in that public goods setting: the seller makes a take-it-or-leave-it offer to a community of tenant farmers who are coerced to participate in a Concordance mechanism for purchasing the land. In Section III.A, we discuss more formally the connection between results in these two settings.

[6]Bailey (1994) provided an excellent survey of this strain of the literature.

II Model

There are $N > 0$ (potential) *sellers* i. Each seller owns a plot of land which she privately values at v_i. There is a single (prospective) *buyer* with value b for the *aggregate plot* comprised of all the sellers' individual plots. We thus assume that the buyer views the plots as perfect complements.[7] We let $V \equiv \sum_i v_i$ denote the aggregate value of the *(seller) community* for the aggregate plot. All agents are assumed to be risk-neutral.

We assume that the buyer is informed about both sellers' relative values and the aggregate uncertainty underlying sellers' individual values. The *share* of seller i is defined by

$$s_i(\sigma, \gamma) \equiv E\left[\frac{v_i}{V} \mid \sigma, \gamma\right],$$

where σ and γ are (typically multidimensional) parameters which respectively represent the buyer's knowledge about relative and aggregate seller value uncertainty. While both known to the buyer, σ and γ are not verifiable by the planner. We assume, without loss of generality (given the multidimensional nature of γ), that the buyer's value is a deterministic function of γ, $b = b(\gamma)$, although we suppress the dependence on γ except where necessary for clarity.

Assumption 1. The shares $s_i(\sigma, \gamma) \equiv s_i(\sigma)$ are independent of γ.[8]

Assumption 2. Given the shares $s_i = s_i(\sigma)$, the values $\frac{v_i}{s_i}$ are independent and identically distributed across i, and independent of σ.[9]

Assumption 1 is equivalent to assuming that a best estimate of each seller's share of the total land value can be made independent of the actual aggregate value. In some settings, this estimate may in fact be available to the planner—in such circumstances, s_i is independent of σ as well as γ (or equivalently, only one value of σ is possible).[10]

[7]While this assumption is extreme, our earlier work (Kominers and Weyl, 2012) showed that issues similar to those we consider here arise even if many potential plots are available as long as a large contiguous block is needed.

[8]This assumption establishes the connection between share-incentive compatibility and approximate individual rationality that we present in Lemma 2, and is thus essential to the approximate individual rationality of Concordance procedures. None of our other results depend on it.

[9]All of our results would continue to hold for general joint distributions over these implied community values, subject to Assumption 1, except for Lemma 1 which links bilateral to asymptotic efficiency. Even the assumptions necessary for Lemma 1 can be substantially weakened: Full independence of seller values is not needed; weak mixing conditions would suffice. Indeed, the independence assumption is used in the proof of Lemma 1 only for a variance bound and an application of Chebyshev's inequality. Thus, Assumption 2 is mostly for expositional purposes—it allows us to avoid using joint distribution notation.

[10]In the case of land assembly, individuals' shares of the total assessed market value of to-be-assembled land can be used to determine shares. In the case of a corporate acquisition, meanwhile, existing share holdings can determine shares; this information is typically available to the planner.

Under Assumption 2, sellers' values v_i arise from an underlying γ-conditional distribution of "implied community values" $\frac{v_i}{s_i(\sigma)}$, which represent the values for the entire plot of land (V) that would be implied by seller i's idiosyncratic value v_i if her share of the total value $s_i(\sigma)$ were perfectly accurate. These implied community values $\frac{v_i}{s_i(\sigma)}$ are distributed independently and identically across sellers i, according to a γ-conditional distribution g_γ on $[\underline{V}, \infty)$, which we assume to be smooth and of full support. This construction induces a smooth, full support, γ-conditional *joint* probability density function $\tilde{g}_\gamma$ for the seller value profile $(v_1, \ldots, v_N)$.

In our framework, the seller community uses a *(collective choice) mechanism* $\mathscr{M}$ to determine whether to sell the community plot. The mechanism $\mathscr{M}$ consists of a set $\mathcal{R} \subseteq \mathbb{R}$ of potential *(seller) reports*, an *allocation rule* $P : \mathcal{R}^N \to \in \{0, 1\}$ determining whether sale takes place, and a *transfer rule* $t : \mathcal{R}^N \to \mathbb{R}^N$ specifying transfers to (or from) sellers.

The collective choice mechanism $\mathscr{M}$ is determined by the buyer, in the following *transaction procedure*:

1. The buyer announces
   (a) an offer $o \in \mathbb{R}$ for the community plot and
   (b) seller share parameter $\hat{\sigma}$,

   which determine a collective choice mechanism $\mathscr{M} = \mathscr{M}(o, \hat{\sigma})$ from a family of mechanisms $\mathfrak{M}$.

2. Sellers submit their reports r_i to the mechanism $\mathscr{M}$, and decision $P(r)$ is determined.

3. The transfer payments $t(r)$ are levvied/delivered. At the same time, in the case of sale (i.e. in the case that $P(r) = 1$), the buyer pays o and each seller receives $s_i(\hat{\sigma}) \cdot o$.

The utility of seller i under this procedure, given offer o, share parameter $\hat{\sigma}$, and mechanism $\mathscr{M}$, is assumed to take the form

$$v_i \cdot (1 - P(r)) + s_i(\hat{\sigma}) \cdot o \cdot P(r) + t_i(r);$$

buyer utility takes the form

$$(b - o) \cdot P(r).$$

A transaction procedure is characterized by its family of mechanisms $\mathfrak{M}$; hence, we simply speak of *procedures* $\mathfrak{M}$. In general, we focus on procedures $\mathfrak{M}(\mathscr{M})$ associated to mechanisms $\mathscr{M} = \mathscr{M}(o, \hat{\sigma})$ with a fixed set of potential seller reports $\mathcal{R}$ and parameterized allocation and transfer rules $P(r) = P(o; r)$ and $t(r) = t(o, \hat{\sigma}; r)$.

Note that we restrict our attention to procedures in which the buyer makes a take-it-or-leave-it offer and the sale decision is determined through collective choice among sellers about whether to accept that offer. This restriction has three principal justifications. First, any approach that involves sellers affecting the aggregate sale price (conditional on a sale) would require sellers to form and coordinate beliefs in ways that we believe would likely make the resulting procedure overly complex. Second, all existing procedures that have been proposed or used for aggregation are of the form we consider. Finally, we note that procedures giving the sellers bargaining power cannot overcome the basic impossibility results we try to resolve (Mailath and Postelwaite, 1990; Hellwig, 2003); thus, our restriction does not rule out approaches which could solve holdout without compromising design criteria.

We denote the N-seller instantiation of a mechanism $\mathscr{M}$ by $\mathscr{M}_N$; the purchase and transfer rules of $\mathscr{M}_N$ are maps $\mathcal{R}^N \to \{0,1\}$ and $\mathcal{R}^N \to \mathbb{R}^N$, respectively. With slight abuse of terminology and notation, we speak of properties that hold for all instantiations $\mathscr{M}_N$ of a mechanism $\mathscr{M}$ as "properties of the mechanism $\mathscr{M}$," writing $\mathscr{M}$ in place of the specific instantiations $\mathscr{M}_N$ whenever doing so does not introduce confusion. Similarly, we denote N-seller instantiations of procedures by $\mathfrak{M}_N$, and speak of "properties of procedures $\mathfrak{M}$." As the number N changes, we assume that the marginal distribution of γ (and thus joint distribution of g_γ and $b(\gamma)$) are constant, though the (joint) distribution of σ may depend on N.

Finally, for simplicity, we often present our discussion in terms of the *net transfers*

$$T_i(r) = T_i(o, \hat{\sigma}; r) \equiv s_i(\hat{\sigma}) \cdot o \cdot P(r) + t_i(r).$$

II.A Equilibrium

We say that report profile r is a *(weakly) dominant strategy report profile* under $\mathscr{M}$ if each individual report r_i is a weakly dominant strategy for i under $\mathscr{M}$. That is, r is a weakly dominant strategy report profile under $\mathscr{M}$ if for each seller i,

$$v_i(1 - P((r_i, \hat{r}_{-i}))) + T_i((r_i, \hat{r}_{-i})) \geq v_i(1 - P(\hat{r})) + T_i(\hat{r})$$

for any report profile $\hat{r}$. We say that a mechanism $\mathscr{M} = \mathscr{M}(o, \hat{\sigma})$ is a *dominant strategy implementable* mechanism if at least one dominant strategy report profile $r = r(o, \hat{\sigma})$ exists for each choice of $(o, \hat{\sigma})$.

For a procedure $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ associated to a dominant strategy implementable mechanism $\mathscr{M}$, the buyer's offer and share parameter choice, $(o, \hat{\sigma})$, determine a dominant strategy seller report profile $r^*(o,\hat{\sigma})$.[11] In this case, we may define an *optimal buyer strategy in dominant strategy seller reporting equilibrium* as any choice $(o^*,\hat{\sigma}^*) = (o^*(\mathscr{M},b),\hat{\sigma}^*(\mathscr{M},b))$ which maximizes the buyer's utility,

$$(o^*,\hat{\sigma}^*) \in \operatorname*{argmax}_{(o,\hat{\sigma})} \left((b-o)P\left(r^*(o,\hat{\sigma})\middle|\,\gamma\right)\right).$$

In this context, a *dominant strategy (seller) equilibrium report* under $\mathfrak{M}$ is a reporting profile $r^*(o^*,\hat{\sigma}^*)$ arising under an optimal buyer strategy $(o^*,\hat{\sigma}^*)$.

Similar notions, such as *Bayes-Nash report profiles*, *Bayes-Nash implementability*, and *Bayes-Nash equilibrium (seller) reports* are defined analogously. More generally, when the equilibrium concept is left to be specified elsewhere or is clear from context, we speak simply of *equilibrium report profiles*, *equilibrium implementability*, and *equilibrium reports*. For technical reasons, we consider only equilibrium concepts under which the reports r_i^* of individual sellers i are not correlated with the valuations v_{-i} of other sellers.[12] When discussing mechanisms and procedures in the abstract, we assume that they are equilibrium implementable unless otherwise stated.

II.B Collective Choice Efficiency

We say that mechanism $\mathscr{M}$ is *equilibrium seller-efficient* if, for each equilibrium report profile r^*, the decision rule $P(r^*)$ maximizes seller welfare,

$$(o-V)P(r^*) = \left(\sum_i \left(s_i(\hat{\sigma})o - v_i\right)\right) P(r^*).$$

II.C Global Efficiency

A natural goal is the maximization of the *(equilibrium) efficiency* of a procedure $\mathfrak{M}$,

$$e(\mathfrak{M}) \equiv \frac{E[(b-V)(2P(r^*)-1)]}{E[(b-V)(2\cdot 1_{b\geq V}-1)]},$$

obtained under equilibrium reports r^*. We say that $\mathfrak{M}$ is *fully efficient (in equilibrium)* if $e(\mathfrak{M}) = 1$ (in equilibrium).

For assembly problems, the status quo benchmark is often important. Hence, in our

[11]Note that we implicitly suppose a selection from the *set* of such profiles. In the sequel, we will explicitly state equilibrium reporting profiles whenever multiplicity is an issue.

[12]This restriction plays a supporting role in the proof of Theorem 3. It is not particularly strong—it rules in even permissive solution concepts like interim correlated rationalizability (Dekel et al., 2007).

discussion we typically break efficiency into two components: the *(equilibrium) efficiency gains*,

$$e_+(\mathfrak{M}) \equiv \frac{E[(b-V)P(r^*)1_{b\geq V}]}{E[(b-V)1_{b\geq V}]},$$

and the *(equilibrium) efficiency losses*,

$$e_-(\mathfrak{M}) \equiv \frac{E[(V-b)(1-P(r^*))1_{b<V}]}{E[(V-b)1_{b<V}]}.$$

The former quantity measures the fraction of potential gains from trade achieved; the latter measures the realized fraction of potential losses from inefficient trade. By construction, $e(\mathfrak{M}) = \lambda e_+(\mathfrak{M}) + (1-\lambda)e_-(\mathfrak{M})$, where

$$\lambda = \frac{E[(b-V)1_{b\geq V}]}{E[(b-V)(2\cdot 1_{b\geq V}-1)]}.$$

We say that a procedure $\mathfrak{M}$ is *gains efficient* if $e_+(\mathfrak{M}) = 1$. Analogously, $\mathfrak{M}$ is *loss efficient* if $e_-(\mathfrak{M}) = 1$.

II.D Individual Rationality

The standard individual rationality condition requires that every seller receive compensation that makes her "whole"; that is, she must have the option to be compensated at a level at least equivalent to her valuation in the case of sale. Formally, a mechanism $\mathscr{M}$ is *individually rational* if for any seller i and profile r_{-i} of other sellers' reports, there is some $r_i \in \mathcal{R}$ such that

$$T_i((r_i, r_{-i})) \geq v_i P((r_i, r_{-i})).$$

II.E Financing

We say that a mechanism $\mathscr{M}$ is *self-financing* if $\sum_i t_i \leq 0$, and *budget-balanced* if $\sum_i t_i = 0$; a procedure $\mathfrak{M}$ has these properties if and only if its associated mechanism does. In general, self-financing procedures are highly desirable, as procedures that are not self-financing may be open to fraudulent exploitation through buyer-and-seller collusion. We thus confine our attention to such procedures, although relaxing this requirement may be an interesting direction for future research.

III The Holdout Problem

Mailath and Postelwaite (1990) showed that for *any* individually rational mechanism which does not run a deficit, no trade occurs in the limit as the number of perfectly complementary sellers grows large. Thus, just as with public goods, encouraging assembly in large markets (with positive probability) requires some degree of coercion.

Unfortunately, unlike in public goods settings, traditional coercive solutions such as eminent domain are undermined when (as in our model) buyers are better informed than public authorities: buyers can exploit coercive power to adversely select the assemblies most harmful to sellers (Kaplow and Shavell, 1996). Moreover, coercion may undermine investment incentives unless a fair distribution of costs and benefits among sellers is maintained (de Soto, 2003).

In this section, we formalize and extend the informal discussion of this three-fold problem that we presented in our previous work (Kominers and Weyl, 2012).

III.A The necessity of coercion

Hellwig (2003), extending the main result of Mailath and Postelwaite (1990), showed that assembly is possible in large markets only if either

- it is known with certainty (*ex ante*) that assembly is efficient (i.e. o is outside the support of V) or
- o grows without bound in N.

Because no buyer will ever make an offer above her valuation b, these conditions transfer over to $b \geq o$. But with the distribution of γ constant across N, the joint distribution of g_γ and $b(\gamma)$ is constant in N as well. In that case, clearly optimal buyer strategies only entail offers o^* within the support of V, and these offers cannot grow without bound as N increases. Thus, the classic Mailath and Postelwaite (1990) result for public goods directly extends to our setting.

Theorem 1 (Mailath and Postelwaite, 1990)**.** *If $\mathscr{M}$ is individually rational and self-financing, then the transaction procedure $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ associated to $\mathscr{M}$ achieves none of the gains from trade in the limit as N grows; that is,*

$$\lim_{N \to \infty} e_+(\mathfrak{M}_N) = 0.$$

A number of procedures have been proposed in hopes of alleviating strategic incentives towards holdout without changing agents' property rights or introducing external subsidies

(Kelly, 2006; Grossman et al., 2010). However, Theorem 1 implies that such procedures typically allow assembly with at most a vanishingly small probability.[13] While the Mailath and Postelwaite result is stated formally in the limit, rates of convergence analysis on these settings have shown that the probability of assembly dies exponentially in the number of sellers; Ely (2009) proved this in the case of dominant strategy implementable mechanisms, and our previous work (Kominers and Weyl, 2012) presented a numerical example. This indicates that the qualitative conclusions of Theorem 1 are relevant in populations of the size that is common in land assembly problems.

Thus, some degree of coercion is necessary in order to enable efficient assemblies. Posner (2005) argued that this is why the policy of "eminent domain" in the Fifth Amendment to the United States Constitution allows the government to take "private property [...] for public use," but only after "just compensation" has been paid.

III.B Possibilities for abuse

Coercion, however, raises a concern in holdout settings that is absent in classic public goods problems: assembly is usually initiated by a private party. If the amount the buyer must pay for the assembly systematically understates the true value of the aggregate plot, buyers may have an incentive to initiative wasteful projects. Thus excessive assembly is at least as great a concern as insufficient assembly, especially among takees and their defenders (Castle Coalition, 2009). In fact, this concern has led states to severely curtail the use of eminent domain, thereby restoring the holdout problem (Morton, 2006).

As Kaplow and Shavell (1996) emphasized, even if a planner makes her best *ex ante* estimate of the value of the aggregate plot, this estimate may systematically understate the true value if prospective buyers are privately informed. In the context of single-seller settings, they argue that if the sellers' values are correlated with buyers' values, then the set of buyers willing to purchase will be adversely selected: the very fact that a buyer is willing to purchase land may indicate that the price offered is too low. If this adverse selection is severe, then absolute protection of sellers' right of refusal may be the only way to avoid overwhelming waste—this is a classic economic rationale for property rights protection.

This point extends to our context. In particular, absent input directly from the sellers,

[13]Hellwig (2003) showed that if the total surplus generated by a project grows as the assembly problem does, the conclusion of Theorem 1 can be avoided. Nevertheless, large projects generating surplus that is small in per-seller terms will fail if a private (voluntary and self-financing) procedure is used.

We believe that this is a large part of the reason why coercive solutions to holdout problems are so widespread: Any effective solution to holdout must function in markets with a large number of sellers. Thus, either coercion or an external subsidy is required; some coercion then seems unavoidable, as mechanisms with external subsidies generate opportunities for defrauding the state.

if the Akerlof (1970) condition for market collapse is satisfied, then it is optimal to prohibit any assembly. Formally, suppose that under the planner's priors,

$$E\left[V \mid b(\gamma) \geq x\right] < x \quad \text{for all } x. \tag{1}$$

Then even though there may be many possible beneficial assemblies, any selected assembly price will induce only assemblies that are wasteful. While this condition, analogous to that of Kaplow and Shavell (1996), at first may seem extreme, we note that it is evaluated over *all* potential projects that could be proposed. Given that most property is likely efficiently held, it seems plausible that a condition like (1) could hold.

Possibilities for abuse are likely why many procedures limit the cases under which coercive assembly is permitted, typically requiring substantial research (and a judicial process) before an assembly can go through. To the extent such limits are effective, they are still likely to be arbitrary and costly and thus destroy some gains from efficient assembly. Moreover, misuse of coercive procedures is difficult to police.[14] A more effective approach is to include some form of seller veto, encouraging the sellers themselves to identify and prevent wasteful assembly.

III.C Fairness and distributive challenges

One efficient means of preventing inefficient assemblies, suggested by Plassmann and Tideman (2009), is to run a Vickrey (1961)-Clarke (1971)-Groves (1973) (VCG) mechanism among the buyer and sellers jointly. In such a setup, the buyer and the sellers are asked to submit their values ($r_i = v_i$ and $o = b$), and the land is awarded to the sellers if $R \equiv \sum_i r_i > o$ and to the buyer if $o \geq R$. If a sale occurs, the buyer must pay o, so long as $R > 0$, and each seller i pays $-\left(o - \sum_{j \neq i} r_j\right)$ if $R > o > \sum_{j \neq i} r_j$, and pays 0 otherwise. As Plassmann and Tideman (2009) observed, this approach is fully efficient. Unfortunately, however, it causes dramatic redistribution of resources away from sellers. Individual sellers might either lose their land without any compensation or be forced to pay large amounts to retain their land even if the assembly attempt is entirely frivolous.

A superficially more appealing approach, used historically in England (Hoffman, 1988) and Japan (Minerbi, 1986) and recently advocated by Heller and Hills (2008), is to allow

[14] Eminent domain, for example, limits assembly to cases of compelling public interest, but

1. interest groups can often pass off private projects as publicly beneficial, and
2. as the United States Supreme Court (2005) argued in *Kelo v. City of New London, Connecticut*, some blurring of the definition of "public" use is inevitable because prohibiting coercive private assemblies can create tremendous inefficiency (because of holdout).

sellers to vote on whether to accept assembly at the offer price. Unfortunately, voting procedures invite manipulations by the buyer; this makes voting ineffective at preventing abuse and can lead to unfair redistribution almost as severe as under VCG. To see this, note that under plurality voting procedures, buyers may attempt to concentrate shares of the offer among a bare-pluraity of the sellers, so as to maximize the chance of sale. In addition to redistributing wealth—51% of sellers, say, receive 100% of the offer—this can promote inefficient assemblies.[15] This risk is exacerbated if—as in a number of historical assembly procedures (see, e.g., Hart, 1996)—veto rights are allocated according to sellers' shares, as then assigning large shares to individual sellers increases the chance of sale even more.

Redistribution away from and among sellers is widely viewed as unjust; it has been the source of significant public outcry against coercive procedures in land assembly (Castle Coalition, 2009) and other holdout settings (Rob, 1989; Hazlett, 2005; Heller, 2008). Moreover, economic arguments suggest that redistribution at the cost of sellers' property rights may be undesirable: it adds idiosyncratic risk for which a liquid insurance market is unlikely to be feasible, and may undermine incentives to invest in property (de Soto, 2003).[16] In fact such concerns about fairness and dynamic incentives are likely an important part of why most developed societies maintain property rights in bilateral trade settings rather than expropriating sellers and running second-price auctions to overcome the inefficiencies identified by Myerson and Satterthwaite (1981). Even without concern for investment incentives, issues of fairness and expropriation risk have long been a concern in coercive public goods settings. Wicksell (1896) and Lindahl (1919) argued that individuals should have to pay a share of the costs of a public project corresponding to their share of the benefits it yields. However, Theorem 1 implies that such a payment structure necessarily undermines any significant chance of the good being provided.

IV Market Design Goals

The preceding discussion shows that attempts to reduce holdout face tradeoffs along three dimensions:

1. Some coercion is necessary in order to promote *efficiency*, but

2. coercion encourages abuse unless sellers are granted some form of *veto rights*, and

[15] This is true even if all sellers have the same values $v_i \equiv \bar{v}$: If $N \cdot \bar{v} = V > b \geq o$ but $o \geq .51 \cdot N \cdot \bar{v}$, then the buyer can offer 51% of sellers $\frac{o}{.51 \cdot N}$ each, and guarantee (inefficient) assembly under plurality voting. If sellers' values are not identical, then the buyer can use his private information about relative shares to target offers to low-value sellers, further increasing the chances of sale under voting produres.

[16] Blume et al. (1984) challenged this traditional view of composition improving investment incentives, arguing that pre-taking investments may not be socially valuable because taken land is typically repurposed.

3. these veto rights themselves lead to *distributional concerns.*

A natural approach, in light of these competing challenges, is to set up an objective function quantifying all of the relevant trade-offs and maximize this over the set of all procedures, subject to constraints implied by (dominant strategy or Bayesian) incentive compatibility. Such "optimal" design analysis, along with impossibility results like those described above, has been the main approach of the public goods tradition.[17] The literature on holdout, meanwhile, has predominantly considered historical institutions without extensively analyzing their properties.[18]

In contrast to these two approaches, we follow a growing literature on *market design*, in which desirable properties are advocated, defended, and then shown to be satisfied by a class of procedures.[19] Specifically, we argue that it may be best to encourage procedures that satisfy strong, yet second-best, efficiency, veto rights, and distributional guarantees, instead of procedures that have perfect guarantees along individual dimensions.

We believe that the market design approach has several advantages in the case of holdout. First, formulating a broadly plausible model formalizing trade-offs between the problems of efficiency, property rights, and fairness may be difficult given the disparate nature of the various objectives.[20] Second, the precise nature of the optimal procedure for reducing holdout will almost certainly depend on features of empirical value distributions that are unlikely to be readily estimable.[21] Finally, this approach may help avoid extremely negative outcomes, as in the spirit of the literature on robustness of policy (Hansen and Sargent, 2008).

In the remainder of this section, we formalize and explain our proposed desiderata for holdout-reducing market design. We focus on the criteria that are novel to this paper.

IV.A Efficiency Criteria

The Myerson and Satterthwaite (1981) Theorem implies that basing a transaction procedure $\mathfrak{M}$ on a self-financing mechanism $\mathscr{M}$ that includes a seller veto cannot achieve the full gains

[17]For example, Mailath and Postelwaite characterized the optimal mechanism for public goods *en route* to their impossibility result (Theorem 1).

[18]Proposed approaches include secret purchases (Kelly, 2006), unanimity rules (Grossman et al., 2010), and voting procedures (Heller and Hills, 2008).

[19]This approach has been successfully applied to problems such as spectrum allocation (e.g., Day and Milgrom, 2008), school choice (e.g., Pathak and Sönmez, 2008), kidney exchange (Roth et al., 2004), and combinatorial assignment (Budish, 2011).

[20]This problem may be especially serious because, as Rawls (1993) emphasized in his analysis of liberal political institutions, policymakers often disagree (particularly in the case of fairness) as to *why* specific properties are desirable.

[21]The Wilson (1987) doctrine argues against using mechanisms that depend upon information that may not be common knowledge among agents.

from trade; that is, $\mathfrak{M}(\mathscr{M})$ is not gains efficient ($e_+(\mathfrak{M}) < 1$). As seller vetoes are necessary for preventing abusive assemblies, a reasonable, second-best goal is to limit a mechanism's inefficiency to that implied by the Myerson and Satterthwaite (1981) Theorem.

Formally: Let $\mathscr{M}^{\text{MS}}$ be the take-it-or-leave-it offer mechanism of Myerson and Satterthwaite (1981), that is, the mechanism for bargaining between a buyer and a single seller in which the buyer makes an offer o—which the seller receives in case of sale—and sale occurs if and only if the seller accepts the buyer's offer. The Myerson and Satterthwaite (1981) Theorem shows that $\mathscr{M}^{\text{MS}}$ is not fully efficient, because the buyer must set price, and so profits (in expectation) from making an offer $o < b$.

Our setting embeds an underlying Myerson and Satterthwaite (1981) problem: a bilateral bargain between the buyer and a *single* seller (the community), whose valuation is distributed according to the community value distribution. Thus, a natural design goal is to try to guarantee as much efficiency in trade as this underlying bargaining problem permits.

Definition 1. A procedure $\mathfrak{M}$ is *bilaterally efficient* if $\mathfrak{M}$ achieves the efficiency of a bilateral bargain between a buyer and a *single* seller whose valuation is distributed according to the community value distribution:

$$e_+(\mathfrak{M}) \geq e_+\left((\mathfrak{M}(\mathscr{M}^{\text{MS}}))_1\right)$$

when the (b-conditional) distribution of the single seller valuation under $(\mathfrak{M}(\mathscr{M}^{\text{MS}}))_1$ is the same as that of V under $\mathfrak{M}$.

When the buyer has no aggregate uncertainty about the sellers' valuations, bilateral efficiency implies an additional attractive second-best efficiency criterion.

Definition 2. A procedure $\mathfrak{M}$ is *asymptotically efficient* if

$$\lim_{N\to\infty} e(\mathfrak{M}_N) = 1.$$

Lemma 1. *A bilaterally efficient procedure is asymptotically efficient if there exists an* $M > 0$ *such that* $Ns_i(\sigma) < M$ *with probability* 1 *for all* N *and* $i \in \{1, \ldots, N\}$.

Lemma 1 follows from the law of large numbers: The inefficiency implied by the Myerson and Satterthwaite (1981) Theorem vanishes when the efficiency of trade is certain. Meanwhile, as N grows the total community value $V = \sum_i v_i$—and hence the efficiency or inefficiency of trade, given the buyer's value b—becomes known with (near-)certainty. Thus, the inefficiency of bilaterally efficient procedures vanishes in the limit.

IV.B Veto Rights Criteria

To prevent abuse of procedures, sellers must have some form of property right. We propose as a baseline rule that assemblies should occur only if they are certain to be efficient, that is, when the buyer's offer exceeds the aggregate seller value.[22] We say procedures with this property have no inefficient sales; clearly, any such procedure is loss efficient.

Definition 3. A mechanism $\mathscr{M}$ *has no inefficient sales (in equilibrium)* if, for the equilibrium report profile r^*,

$$(b - V)P(r^*) \geq 0.$$

We say that a procedure $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ *has no inefficient sales (in equilibrium)* if its underlying mechanism $\mathscr{M}$ has no inefficient sales.

IV.C Fairness Criteria

As our discussion of the Plassmann and Tideman (2009) VCG implementation (Section III.C) illustrated, preventing abusive purchases is not on its own sufficient to avoid harming sellers—it is also important to ensure that the sellers receive fair compensation in case of sale. While we have discussed our procedures using notation $T_i(r) = s_i(\hat{\sigma})oP(r) + t_i(r)$ suggesting that sellers are entitled to their shares of the offer, an explicit criterion is required in order to give this notation normative meaning. An appealing approach is to guarantee sellers that, if they are willing to abstain from influencing the collective decision, they can guarantee themselves their shares of the offer (conditional on sale) without having to make any payments.

Definition 4. A procedure $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ *guarantees seller i her fair share (in equilibrium)* if for the equilibrium report profile r^*, there exists $r_i \in \mathcal{R}$ such that

1. $P((r_i, r^*_{-i})) = \tilde{P}(\tilde{r}^*_{-i})$, where $\tilde{P}$ is the decision rule of $\tilde{\mathscr{M}} = \mathscr{M}_{N-1}((1 - s_i(\hat{\sigma}))o, \tilde{\sigma})$, $\tilde{\sigma}$ is chosen so that the share of each seller $j \neq i$ under $\tilde{\mathscr{M}}$ is $\frac{s_j(\hat{\sigma})}{1 - s_i(\hat{\sigma})}$, and $\tilde{r}^*_{-i}$ is the equilibrium report under $\tilde{\mathscr{M}}$,[23] and
2. $t((r_i, r^*_{-i})) \geq 0$.

We say that a procedure that has this property for each seller i *guarantees sellers their fair shares (in equilibrium).*

[22] A rational buyer never offers more than his value in equilibrium. Thus, assuming that the buyer is rational, we have $o^* \leq b$ in equilibrium; hence, we know that $b \geq V$ whenever $o^* \geq V$.

[23] We have not made any formal assumptions on the distribution of σ; hence, it is not technically guaranteed that an appropriate value of $\tilde{\sigma}$ exists. We use this notation here only as a shorthand for the fact that $\mathscr{M}_{N-1}((1 - s_i(\hat{\sigma}))o, \tilde{\sigma})$ must treat sellers $j \neq i$ as having the same relative shares as under $\mathscr{M}_N(o, \hat{\sigma})$.

The first condition of Definition 4, while somewhat cumbersome to state, simply formalizes our notion of abstention: if i submits the distinguished report r_i when other sellers report according to the equilibrium, then the decision is made as if i were not in the community. The second condition of Definition 4 ensures that in this case, seller i is guaranteed to receive at least her share of the offer conditional on sale and loses nothing if sale does not occur.

Note that the fair share guarantee only ensures that sellers i receive at least their *buyer-reported* shares $s_i(\hat{\sigma})$. Thus, the guarantee achieves our normative goal only if buyers truthfully reveal their private information about shares. Moreover, as our discussion in Section III.C highlighted, outcome efficiency may also depend on truthful share revelation. Thus, we seek procedures under which truthful reporting of the share value parameter σ is optimal for buyers. This is particularly important in contexts such as land assembly, where the purchaser may be the government and thus may have broad scope to determine the allocation of shares among individual sellers.

Definition 5. A procedure $\mathfrak{M}$ is *share-incentive compatible (in equilibrium)* if, for all buyer valuations b, there is some optimal buyer strategy under $\mathfrak{M}$ (in reporting equilibrium) of the form (o^*, σ), that is, if the buyer always (weakly) prefers to reveal σ truthfully.

Share-incentive compatibility requires only that buyers have no incentive to misreport shares—it does not guarantee that buyers have a strong (or even a positive) incentive to report them correctly. However, many features of typical holdout settings may provide some stricter incentive for truthful share revelation.[24] Share-incentive compatibility can then be seen as guaranteeing that this "natural" incentive will not be swamped by poor incentives created by the choice of transaction procedure.

When combined with seller-efficiency and the fair share guarantee, share-incentive compatibility ensures an attractive "approximate property rights" condition.

Definition 6. A procedure $\mathfrak{M}$ is *approximately individually rational for seller i (in equilib-*

[24]Political institutions governing holdout situations typically make transactions in which few individuals consent (or significantly oppose sale) more costly for buyers. Thus, buyers may report shares correctly, in an attempt to minimize opposition. This informal institution could be formalized into any share-incentive compatible procedure (at some efficiency cost) by requiring unanimous consent for a sale with small probability. (If the buyer may invest in improving his knowledge of the shares, the optimal level of this probability might be chosen to trade off efficiency against fairness.)

Moreover, if sellers exhibit a tendency to pay more to stop disadvantageous sales than to achieve advantageous sales—a plausible kind of loss aversion—the seller will have an incentive to truthfully report shares (given share-incentive compatibility) if the choice mechanism is efficient, as this will reduce the number of "losers" and amount of "loss," thereby improving the chance of sale.

Finally, the buyer may, as posited by Carroll (2011), have a small inherent preference for truth-telling.

rium) if, for the equilibrium report profile $r^* = r^*(o^*, \hat{\sigma}^*)$, there exists $r_i \in \mathcal{R}$ such that

$$T_i((r_i, r^*_{-i})) \geq E[v_i \mid \sigma, \gamma] \cdot P((r_i, r^*_{-i})).$$

We say that $\mathfrak{M}$ is *approximately individually rational (in equilibrium)* if $\mathfrak{M}$ is approximately individually rational for each seller i.

Our fair share and approximate individual rationality conditions only guarantee a level of compensation at a particular distinguished value of r_i—not necessarily at the r^*_i chosen by sellers i in equilibrium. However, the availability of the distinguished r_i places a lower bound on seller i's utility in equilibrium: by revealed preference, i must do no worse than if she allows others' preferences to determine whether a sale occurs and receives an unbiased estimate of her value in case of sale. This guarantee is inspired by, and extends to the holdout setting, the Bergstrom (1979) proposal that in public goods settings each individual should pay no more than an unbiased estimate of her valuation based on all available aggregate information.[25]

Requiring individual rationality is essentially equivalent to imposing "perfect preservation of property rights," using the absolutist conception of property established in the Anglo-Saxon legal tradition (Dana and Merrill, 2002). However, as Theorem 1 illustrates, fully preserving individual property rights grossly inconsistent with *ex post* social efficiency (with any trade at all in the limit). As a result, practical procedures for solving the holdout problem must abrogate some property rights. As also discussed by Bergstrom (1979), our approximate individual rationality guarantee follows a Continental tradition which emphasizes the importance of socially fair sharing of the burdens of social projects rather than strict protection of individual property rights.

Lemma 2. *If $\mathfrak{M}$ is seller-efficient, guarantees sellers their fair shares, and is share-incentive compatible, then $\mathfrak{M}$ is approximately individually rational.*

Proof. As $\mathfrak{M}$ is share-incentive compatible and we are concerned with equilibrium behavior, we take $\hat{\sigma} = \sigma$ in the sequel. Now, as $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ guarantees seller i her fair share, there is some $r_i \in \mathcal{R}$ satisfying the conditions of Definition 4. As $\mathfrak{M}$ is seller-efficient in equilibrium, we know that sale occurs under $\tilde{\mathscr{M}}$ under report $\tilde{r}^*_{-i}$ only if

$$(1 - s_i(\sigma))o \geq \sum_{j \neq i} v_j.$$

[25] As there is uncertainty about the the aggregate shock γ, the approximate individual rationality guarantee is more appealing than an estimate which neglects the buyer's and sellers' information about γ.

It follows that if sale occurs under $\mathscr{M}$ $(1 = P((r_i, r^*_{-i})) = \tilde{P}(\tilde{r}^*_{-i}))$, then

$$o \geq \frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)}, \tag{2}$$

so that i receives at least

$$T_i((r_i, r^*_{-i})) = s_i(\sigma)o + t_i((r_i, r^*_{-i})) \geq s_i(\sigma)\frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)} + t_i((r_i, r^*_{-i})) \geq s_i(\sigma)\frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)},$$

where the first inequality follows from (2) and the second inequality follows from the second condition (of Definition 4) on r_i.

It thus suffices to prove that $s_i(\sigma)\frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)}$ is an unbiased estimator of v_i. To see this, we note that regardless of γ, our assumptions on the structure of sellers' valuations imply that the value $\frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)}$ is an unbiased estimator of V:

$$\begin{aligned} E\left[s_i(\sigma)\frac{\sum_{j \neq i} v_j}{1 - s_i(\sigma)} \;\middle|\; \sigma, \gamma\right] &= E\left[s_i(\sigma)\frac{\sum_{j \neq i} s_j(\sigma)V}{1 - s_i(\sigma)} \;\middle|\; \sigma, \gamma\right] \\ &= E\left[s_i(\sigma)V \mid \sigma, \gamma\right] \\ &= E\left[v_i \mid \sigma, \gamma\right]. \end{aligned}$$

□

V Concordance Procedures

Our approach to reducing holdout is inspired by Cournot's solution to the collaboration problem. Cournot (1838) argued that producers of complements should merge so as to fairly share in—and hence internalize—each others' profits.[26] We see this suggestion, as it applies to holdout, as consisting of two parts:

1. Sellers should divide profits from a sale according to a pre-specified formula, just as a merger divides stock in the conglomerate among the shareholders of the merging firms.

2. Sellers should be incentivized to share information by paying for externalities caused when influencing the group decision, just as divisions of a firm (Groves and Loeb, 1979) are incentivized to communicate with headquarters.

Our *Concordance* procedures follow this simple rubric: The sale decision is determined using an efficient collective choice mechanism. The mechanism itself is constructed so as

[26]Lehavi and Licht (2007) also suggest the notion of a "merger" in abstract terms, but without providing formal analysis or an explicit transaction procedure.

to guarantee sellers the option of "exerting no influence" on the sale decision by reporting $r_i = 0$. Exerting no influence guarantees a seller her fair share.

Definition 7. Formally, an equilibrium implementable mechanism $\mathscr{M}$ is a *Concordance mechanism* if

1. it is self-financing,
2. it is seller-efficient in equilibrium, and
3. $r_i = 0 \implies t_i(r) \geq 0$.

A procedure $\mathfrak{M}(\mathscr{M})$ associated to a Concordance mechanism $\mathscr{M}$ is called a *Concordance procedure.*

Equilibrium seller-efficiency—the second condition of Definition 7—implies that, in equilibrium, the buyer's optimal strategy is to make the the monopsonist-optimal offer

$$o^* = \text{argmax}_o (b - o)\overline{G}_\gamma(o),$$

where $\overline{G}_\gamma$ is the γ-conditional cumulative distribution function of V. Thus, in equilibrium, the outcome of any Concordance procedure is identical to that of a bilateral bargain between the buyer and a single "community seller," with the distribution of the community seller's value being that of $V = \sum_i v_i$. These observations immediately prove the following result.

Lemma 3. *Procedures associated to equilibrium seller-efficient mechanisms have no inefficient sales and are bilaterally efficient.*

Lemma 3 implies loss efficiency, and together with Lemma 1 yields asymptotic efficiency, as well.

Theorem 2. *Concordance procedures have no inefficient sales, are bilaterally efficient and loss efficient, and are asymptotically efficient under the conditions of Lemma 1.*

Theorem 2 shows that Concordance procedures alleviate holdout: Even as the number of sellers grows large, the efficiency of Concordance procedures does not deteriorate below that of bilateral trade. Moreover, Concordance procedures become fully efficient in the limit, provided that seller values are somewhat independent.

Lemma 4. *Procedures associated to equilibrium seller-efficient mechanisms are share-incentive compatible.*

Lemma 4 shows immediately that Concordance procedures are share-incentive compatible. It follows from this and the third condition of Definition 7 that Concordance procedures guarantee sellers their fair shares. Finally, combining these observations with Lemma 2 shows that Concordance procedures are approximately individually rational.

Theorem 3. *Concordance procedures are share-incentive compatible, guarantee sellers their fair shares, and are approximately individually rational.*

The third condition of Definition 7 ensures that payoff guaranteed by approximate individual rationality is particularly simple for sellers i to achieve: it arises under the report $r_i = 0$.

VI Example Concordance Procedures

VI.A VCG Concordance

In Section III.C, we illustrated how a standard implementation of VCG can redistribute seller resources unfairly. Meanwhile, Theorem 1 implies that a fully individually rational version of VCG would not be self-financing in our setting. However, the Concordance approach provides an alternative normalization of VCG payments that preserves approximate individual rationality, giving rise to a natural dominant strategy implementable Concordance procedure.

In this procedure, sale occurs if $R \geq 0$, that is if sellers in aggregate favor sale *given that they receive their shares of the offer ($s_i(\hat{\sigma})o$) if the sale takes place*. Thus while sellers do not have "property rights" over v_i as under full individual rationality, they do have property rights over their shares of the offer conditional on a sale. Taxes are levied on sellers who are pivotal in the sense that R and $R_{-i} \equiv \sum_{j \neq i} r_j$ are on different sides of 0; pivotal sellers i pay a tax equal to the harm caused, $|R_{-i}|$.

Formally, VCG Concordance (VCGC) is the procedure associated to the mechanism with $\mathcal{R} = \mathbb{R}$, $P(r) = 1_{R \geq 0}$, $t_i(r) = -1_{R_{-i} \cdot R < 0} |R_{-i}|$.

It follows from well-known results that VCG is self-financing, and that under VCG sellers report their (positive or negative) surplus from sale,

$$r_i^* = s_i(\hat{\sigma})o - v_i,$$

in dominant strategy equilibrium. The report $r_i = 0$ then corresponds to indifference, and avoids tax payments with certainty: $t_i(0) = 1_{R_{-i} \cdot R < 0} |R_{-i}| = 0$ since $R_{-i} = R$ when

$r_i = 0$. These observations show that VCG is a dominant strategy implementable Concordance mechanism. Thus, VCGC is a Concordance procedure for which the guarantees stated in Theorems 2 and 3 hold whenever sellers act rationally in their own interests. This "straightforwardness" for sellers is a crucial advantage of VCGC over other Concordance procedures. These observations are summarized in the following proposition.

Proposition 1. *VCGC is self-financing, has no inefficient sales, and is bilaterally efficient, loss efficient, asymptotically efficient under the conditions of Lemma 1, share-incentive compatible, and approximately individually rational in dominant strategy equilibrium.*

The main result of Green and Laffont (1977) shows that only Groves mechanisms can be seller-efficient in dominant strategy equilibrium. This characterization leads to the following uniqueness result for VCGC.

Proposition 2. *VCGC is the unique self-financing procedure that is seller-efficient and guarantees sellers their fair shares in dominant strategy equilibrium.*

Unfortunately, because of its reliance on the VCG mechanism, VCGC has two drawbacks. First, VCGC is highly vulnerable to collusion among small groups of sellers: there are equilibria in which any two sellers can, at zero cost to themselves, completely determine whether sale is approved.[27] Second, VCGC is not budged-balanced; its revenues must be destroyed or given to a third-party in order to preserve VCG's incentive properties. This either offsets some of the efficiency benefits of the Concordance approach as the resources must be destroyed or, if they are not and are instead given to a third-party, like a federal government, may invite exploitation by that party.

VI.B Expected Externality

It is well-known that opportunities for collusion may be reduced, and the budget may be balanced, by having sellers pay their expected, rather than realized, externalities (Arrow, 1979; d'Aspremont and Gérard-Varet, 1979). However, implementing an "expected externality mechanism" requires the mechanism designer to have knowledge of the distribution of seller valuations and depends heavily on the beliefs of agents, violating the Wilson (1987) doctrine. Especially the first feature makes it difficult to implement in most settings. Nevertheless, examining the Concordance procedure associated to the expected externality mechanism

[27] This problem is well-known to be particularly severe if, as seems likely in applications like corporate acquisitions, there is a very large number of sellers and sellers can easily "de-merge," splitting one individual into two, each with half the share, who can then express identical, extreme preferences (Ausubel and Milgrom, 2005).

helps frame the connection between VCGC and the more practical procedure we describe in the next section.

We assume throughout this section that the planner can verify γ, and that the valuations v_i are independent of b and of one another, conditional on γ.[28] Given these assumptions, the planner can calculate, for any r_i, the expected Pigouvian tax the seller would have to pay under VCGC in equilibrium. Indeed, this is just

$$\mathrm{EE}_i(r_i) \equiv E_{v_{-i}}\left[|R_{-i}| 1_{(r_i+R_{-i})\cdot R_{-i}<0} \mid \gamma\right],$$

where within the expectation $R_{-i} = \sum_{j\neq i}(s_j(\hat{\sigma})o - v_j)$ takes its dominant strategy equilibrium value.

It is well-known that mechanisms in which sellers pay their expected externalities are Bayes-Nash incentive compatible. This intuition leads to the *Expected Externality Concordance (EEC)* procedure, based on the expected externality mechanism with $\mathcal{R} = \mathbb{R}$, $P(r) = 1_{R\geq 0}$, and

$$t_i(r) = -\underbrace{\mathrm{EE}_i(r_i)}_{\text{Pigouvian tax}} + \underbrace{s_i(\hat{\sigma})\sum_{j\neq i}\frac{\mathrm{EE}_j(r_j)}{1-s_j(\hat{\sigma})}}_{\text{tax refund}}.$$

The fact that the expected externality mechanism is actually a Concordance mechanism is immediate because if $r_i = 0$ then $(r_i+R_{-i})R_{-i} = (R_{-i})^2 \geq 0$, so that $t_i(r) \geq 0$. While EEC is not implementable in dominant strategy equilibrium, it is implementable in *Bayes-Nash equilibrium*, with the same reporting strategy as for VCGC:

$$r_i^* = s_i(\hat{\sigma})o - v_i.$$

EEC is budget-balanced, as

$$\sum_j \mathrm{EE}_j(r_j) = \sum_j\left(\sum_{i\neq j}\frac{s_i(\hat{\sigma})}{1-s_j(\hat{\sigma})}\right)\mathrm{EE}_j(r_j) = \sum_i s_i(\hat{\sigma})\sum_{j\neq i}\frac{\mathrm{EE}_j(r_j)}{1-s_j(\hat{\sigma})}.$$

Finally, just as in the case of VCGC, the buyer's outcome under EEC is independent of the allocation of shares; this implies share-incentive compatibility.

Proposition 3. *EEC is budget-balanced, has no inefficient sales, and is bilaterally efficient, loss efficient, asymptotically efficient under the conditions of Lemma 1, share-incentive compatible, and approximately individually rational in Bayes-Nash equilibrium.*

[28] This is without loss because, as Cremer and Riordan (1985) showed, the implementation we desire can be incentive-compatibly delegated to the buyer in the case that only she knows γ.

Relative to VCGC, EEC trades straightforwardness for budget balance. The expected externality mechanism is the unique budget-balanced, seller-efficient mechanism which is implementable in Bayes-Nash equilibrium and guarantees that

$$\frac{t_i}{t_j} = \frac{s_i(\hat{\sigma})}{s_j(\hat{\sigma})} \quad \text{when } r_i = r_j = 0. \tag{3}$$

Thus, like VCGC, EEC is essentially unique in achieving its package of properties.

Proposition 4. *EEC is the unique budget-balanced Concordance procedure which is implementable in Bayes-Nash equilibrium and guarantees* (3)*.*

VI.C Quadratic Vote Buying

While VCGC and EEC are natural Concordance procedure implementations, they each suffer from drawbacks: VCGC is susceptible to collusion and is not budget balanced, while EEC requires excessive amounts of information about the distribution of seller valuations. VCGC may still be attractive, given its other benefits—especially when the number of sellers is relatively small, so that collusion is likely to be detected. However, as Theorem 1 illustrates, holdout problems are most severe when the number of sellers is *large*. Luckily, Weyl (2012) has shown that central limit-type results imply that in large markets we can implement a mechanism—*Quadratic Vote Buying (QVB)*—which is similar to the expected externality mechanism but does not require direct knowledge of distributional details.

In QVB, individuals purchase votes at quadratic cost, decisions are made based on the majority of purchased votes and all revenues are then returned to the seller community. As with the mechanisms discussed above, this can easily be adapted to a procedure for reducing holdout. As before $\mathcal{R} = \mathbb{R}$ and $P(r) = 1_{R \geq 0}$. "Votes" for (positive) or against (negative) sale given offer o are sold at quadratic cost, which is refunded (to other sellers) as in the expected externality mechanism:

$$t_i(r) = -\underbrace{r_i^2}_{\text{cost of vote purchase}} + \underbrace{s_i(\hat{\sigma}) \sum_{j \neq i} \frac{r_j^2}{1 - s_j(\hat{\sigma})}}_{\text{revenue refund}}.$$

Clearly, QVB is budget-balanced and allows sellers i to "exert no influence" on the sale decision by choosing $r_i = 0$. Upon exerting no influence, i is guaranteed that $t_i(r) \geq 0$. And the revenue refunds are constructed (uniquely) so that, as in EEC, (3) holds—sellers exerting no influence receive refunds in proportion to their shares. Nevertheless, QVB is not precisely a Concordance mechanism as it is not seller-efficient for any finite N: the QVB mechanism *approximates* the expected externality mechanism increasingly well as the population grows large.

Thus, QVB should be interpreted as an "approximate" Concordance mechanism. Results of Weyl (2012) show that

1. the inefficiency of QVB shrinks at rate $O(\frac{1}{\sqrt{N}})$,
2. seller coalitions small relative to the total number of sellers cannot significantly reduce QVB's efficiency or the revenues it raises through manipulation, and
3. collusion never occurs in equilibrium (the more effective collusion is at reducing efficiency, the greater the incentives for unilateral deviation),

at least in the case that $s_i(\hat{\sigma})o - v_i$ is i.i.d. with mean 0. Furthermore, Goeree and Zhang (2012) have shown experimentally and computationally that, for some common distributions of values, QVB achieves impressive efficiency even in small populations. While these results are special and the more general analysis of Weyl (2012) is very much in progress, we think it likely that QVB gives a promising approximate Concordance implementation for use in the large markets most plagued by holdout.

VI.D Comparison to plurality voting

The most common collective choice mechanism for binary decisions is *(plurality) voting*, either *unweighted* or *weighted by shares*. Formally, in a voting mechanism, $t_i(r) = 0$ for all i and r, and for some pre-specified threshold X, either

$$P(r) = 1_{\left(\sum_{i:r_i \geq 0} 1\right) > N \cdot X}$$

(for unweighted voting) or

$$P(r) = 1_{\left(\sum_{i:r_i \geq 0} s_i(\hat{\sigma})\right) > X}$$

(for weighted voting). For example, taking $X = .5$ gives rise to the standard majority rule system. Voting mechanisms have been widely used in holdout settings historically; unweighted voting was used for land assembly in England (Hoffman, 1988) and Japan (Minerbi, 1986) while weighted voting was used for a similar purpose in the American colonies (Hart, 1996) and in corporate acquisitions.[29] Bierbrauer and Hellwig (2011) have shown that only voting mechanisms are dominant strategy implementable and collusion-proof in large markets, and Heller and Hills (2008) have advocated the use of voting mechanisms in transaction procedures.

[29] Note also that the standard eminent domain mechanism arises as the voting mechanism with $X = 0$.

Because voting mechanisms have $t_i \equiv 0$, they trivially satisfy the third condition of Definition 7. However, voting mechanisms are typically inefficient in equilibrium. In large populations, they are efficient only when the population quantile X in the distribution of implied community values is equal to its mean (Bowen, 1943; Ledyard and Palfrey, 1994, 2002), and in small populations efficiency is not guaranteed even in this case. In addition to eliminating the efficiency guarantees of Concordance, this also undermines voting mechanisms' fairness properties: as we discussed in Section IV.C, voting mechanisms incentivize buyers to misrepresent sellers' shares.

Thus, plurality voting is not even a Concordance mechanism in an approximate sense. Instead, the inefficiency and unfairness of voting-based procedures stand in stark contrast to the more desirable properties of VCGC, EEC, and the "approximate Concordance" procedure associated to QVB.

VII Applications

We framed most of our preceding discussion in terms of land assembly, to simplify the exposition and make clear the applicability of our framework. However, land assembly is only one of several holdout settings to which our solutions are applicable. In previous work (Kominers and Weyl, 2012), we detailed an application to the Federal Communications Commission's ongoing project to reassemble fragmented spectrum. Now, we discuss two other applications: corporate acquisitions and patent pool formation.[30]

VII.A Corporate acquisitions

When one individual or corporation seeks a controlling share in a public firm, most countries require that it make a bid for all shares (Kirchmaier et al., 2009), which are typically controlled by a wide group among the public.[31] However, because individuals have heterogeneous risk-aversion and belief-driven infra-marginal utility from investing in the to-be acquired firm, it would be nearly impossible for a prospective buyer to purchase all shares voluntarily. Thus to allow acquisitions to take place, nearly every jurisdiction uses a voting

[30]Other examples of holdout abound: Rules in most countries require the consent of a supermajority of creditors to a debt renegotiation outside of bankruptcy, with thresholds differing across countries (La Porta et al., 1998). Class action legal settlements are often plagued by holdouts (Rob, 1989). Heller (2008) surveyed a variety of other examples, from post-Communist property transitions in eastern Europe to share-cropping relations in the post-Bellum South.

[31]These regulations are designed to protect minority shareholders' interests in the case of takeovers by firms whose interests do not concord with strict divisional profit maximization, and to help reduce free-riding on corporate efficiency improvements (Grossman and Hart, 1980).

procedure, allowing consent by some super-majority of shareholders to squeeze out (Croft and Donker, 2006) or overrule (Armour and Skeel, 2007) the remaining holdouts.

In corporate acquisitions settings, ruling out inefficient sales helps protect shareholders' collective investment incentives and prevents exploitation by raiders. Meanwhile, guaranteeing each seller a share-weighted fraction of the collective settlement (as under the approximate individual rationality guarantee) corresponds to paying individuals their shares of the (fair) acquisition price, as is mandated by various shareholder protections in many jurisdictions (Shleifer and Vishny, 1997). Although these "shares" are apparently public, share-incentive compatibility is still valuable if the buyer can make side-payments to some sellers. Shleifer and Vishny (1997) surveyed historical cases of expropriation when manipulating shares may change the outcome of the election: for example, a purchaser may secretly control or bribe a bare majority and expropriate remaining minority holders. The possibility of such manipulations have become more severe in recent years because of the decoupling of sellers' voting and economic rights through derivatives markets. Decoupling changes sellers' proceeds from sale, thus enabling exploitation of voting mechanisms; it can also cause other inefficiencies, as in recent instances of "empty voting" and "hidden ownership" (Hu and Black, 2005, 2007, 2008; Barry et al., 2012). Using seller-efficient mechanisms as in Concordance eliminates the possibility of such inefficient exploitation.[32]

VII.B Patent pool formation

As Lerner and Tirole (2004) discussed, investors often assemble pools of complementary patents and license them jointly to avoid the complements problem identified by Cournot (1838). However, the holdout problems that arise in patent pool formation can depress both returns to the original inventors and follow-on innovation (Heller and Eisenberg, 1998). In standard patent pool arrangements, pools are formed only by universal consent, creating precisely the holdout problems we have considered whenever individuals have different beliefs about the marginal value of their own patents outside the pool.

While coercive mechanisms have not, to our knowledge, been used in patent pool formation in the past, concerns over the proliferation of patent thickets have led economists such as Shapiro (2001) to consider coercive assembly. However, most are wary of undermining investment incentives through potentially abusive coercion. Our approximate property rights guarantees provide a natural bulwark against such abuse. Share-incentive compatibility is particularly important in this context because, unlike in land assembly where real

[32]Work in progress by Weyl and Eric Posner will examine in greater detail how using seller-efficient mechanisms, and particularly QVB, eliminates the feasibility of such manipulations and the need for explicit minority shareholder protections.

estate agents can often generate reasonable approximations, there are few natural external agents to assess the relative marginal contributions of each patent and thus the pool assembler/organizer will likely play a large role in determining shares in any assembly.[33] Patent reform has been an important part of the high-tech policy agenda in recent years, as evidenced by the United States Patent Reform Act of 2011. Patent pool formation is thus a natural area for the application of new, more efficient approaches.

VIII Conclusion

In this paper we have introduced a market design framework for reducing holdout in applications like land assembly, patent pool formation, and corporate acquisitions. We proposed a collection of properties which balance efficiency, fairness, and implementability, along with a class of procedures that achieve our desiderata.

In our Concordance procedures, the prospective buyer makes a take-it-or-leave it offer to the sellers, and the sellers use an efficient collective choice mechanism to decide as a group whether to accept the buyer's offer. Key to our approach is a simple "pivot": the buyer's offer is divided among the sellers according to *shares*, so as to be independent of individual sellers' actions. Each seller retains the option of receiving her share of the offer in exchange for exerting no influence on the collective decision. This approach allows us to guarantee as much efficiency as would be achieved in a bilateral bargain between the buyer a single seller representing the community. At the same time, it provides a strong individual rationality guarantee: each seller can ensure herself at least an unbiased estimate of her value.

Extensions of our procedures could expand their ranges of applicability. Concordance procedures place full property rights into community hands, but it would be simple—and natural in land assembly contexts—to place property rights partially into the buyer's hands; it is known that this helps mitigate the efficiency distortions introduced by bilateral bargaining (Segal and Whinston, 2011). Our Concordance procedures all require sellers to make payments to enforce true community revelation about the preference for sale. In the real world, sellers often face budget constraints that may make uncertainty in these payments (as under VCG) unattractive. Thus, pratical implementation in many settings will likely favor approaches like QVB.[34]

[33] Antitrust authorities typically prohibit pools from including mutually substitutable patents within a single pool. However complementarity is rarely perfect and some patents have higher underlying values than others outside a pool, deriving, e.g., from broader applicability.

[34] Designs (similar to those of Pai and Vohra (2011) for auctions) that come close to preserving the properties of Concordance procedures while accommodating bidders with privately known budget constraints would be a challenging but practically important extension of our work.

The necessity of coercion for holdout reduction also leads to questions about how relaxations of individual rationality affect sellers' investment incentives. Effective alternatives to a community veto as a means of avoiding frivolous assembly would also be valuable.

Finally, we note that we restricted our attention to a case of perfect complements, assuming away competition between aggregate land plots. In other work (Kominers and Weyl, 2012), we analyzed *combinatorial holdout*, in which competition across plots can (sometimes) partially or fully substitute for coercion. It is possible that a sophisticated procedure, perhaps resembling an auction or exploiting overlaps in cluster membership, would make competition across plots even more powerful. In any case, determining efficient and practical procedures for fully general holdout settings is an important open design problem.

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Appendix: Proofs Omitted from the Text

Proof of Lemma 1

Given that γ is known to the buyer, we consider the analysis for any given γ, suppressing the dependence thereon. (Given that the result holds for all γ, it must hold on average across γ values.) Throughout this proof, we use *superscripts* to index variables and functions that vary with N.

We let μ and ξ^2 respectively represent the mean and variance of the i.i.d. process generating $x_i^N \equiv \frac{v_i^N}{s_i^N(\sigma)}$. As $V^N \equiv \sum_{i=1}^N v_i^N = \sum_{i=1}^N x_i^N s_i^N$ and $\sum_{i=1}^N s_i^N = 1$, we have $E[V^N] = \mu$ and $\text{Var}[V^N] < \frac{M^2\xi^2}{N}$, by the share bound and i.i.d. hypotheses. We denote the density of V^N by f^N.

Because the gains from trade are bounded away from zero, it suffices to demonstrate that the total inefficiency of $\mathfrak{M}_N$ vanishes as $N \to \infty$.

Now, a buyer's offer choice is equivalent to the selection of a probability of sale. Thus, we may interpret the buyer's maximization problem as the selection of probability of sale q in

$$q(b - S^N(q)), \tag{4}$$

where $S^N(q) = (F^N)^{-1}(q)$ denotes inverse supply. We let $\tilde{q}^N(b)$ be the optimal choice of q in (4), for a buyer with value b.

The buyer always offers $o \leq b$, hence inefficient sales will never occur. Thus, the total inefficiency of $\mathfrak{M}_N$ is given by

$$\int_0^\infty \int_{\underline{V}}^\infty (b - V) 1_{o^N(b) < V} f^N(V) h(b) \, dV \, db \leq \int_0^\infty b[F^N(b) - \tilde{q}^N(b)] h(b) \, db, \tag{5}$$

where $o^N(b)$ is the optimal offer of a valuation-b buyer facing N sellers, and h is the density of b.

By the one-sided Chebyshev inequality, we have for any $\alpha > 0$,

$$\text{Prob}[V^N - \mu \geq \alpha] \leq \frac{M^2 \xi^2}{M^2 \xi^2 + N \alpha^2},$$

which vanishes as $N \to \infty$. It follows that $S^N(q) \to \mu$ as $N \to \infty$ (pointwise). Thus, for any fixed $b > \mu$ and (sufficiently small) $\epsilon > 0$, we have

$$(q + \epsilon) \left(b - S^N(q + \epsilon)\right) > q(b - S^N(q)),$$

for N sufficiently large (as $(S^N(q + \epsilon) - S^N(q)) \to 0$). Thus, $\tilde{q}^N(b) \to 1$; it then follows that the right side of (5) vanishes as $N \to \infty$ for all $b > \mu$, as we always have $F^N(b) \geq \tilde{q}^N(b)$.[35]

Meanwhile, again by the Chebyshev inequality, we have $F^N(b) \to 0$ for any fixed $b < \mu$. It then follows that the right side of (5) vanishes as $N \to \infty$ for all $b < \mu$; combining this with our previous observations (and the fact that $b = \mu$ with probability 0) proves the result.

Proof of Lemma 4

Under a procedure $\mathfrak{M}$ associated to a seller-efficient mechanism, sale occurs in equilibrium if and only if

$$o^* \geq V. \tag{6}$$

As the share parameter $\hat{\sigma}$ does not affect whether (6) holds, the buyer is indifferent among share parameter reports $\hat{\sigma}$ under $\mathfrak{M}$ (in equilibrium). It follows that (o^*, σ) is an optimal buyer strategy (in equilibrium); hence, $\mathfrak{M}$ is share-incentive compatible.

[35] This follows from an application of the dominated convergence theorem to (5); such an application is valid because we must have $\int_0^\infty b h(b) \, db < \infty$ (in order for the efficiency of $\mathscr{M}_N$ to be well-defined) and $0 \leq F^N(b) - \tilde{q}^N(b) \leq 1$.

Proof of Theorem 3

Lemma 4 shows that any Concordance procedure $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ is share-incentive compatible. We thus take $\hat{\sigma} = \sigma$.

Claim. *Under $\mathfrak{M}$, if r^* is an equilibrium report profile and $s_i(\sigma)o - v_i = 0$, then $(0, r^*_{-i})$ is also an equilibrium report profile.*

Proof. We consider any equilibrium report profile r^* and suppose that $r^*_i \neq 0$ for some seller i. The utility of seller i under equilibrium report profile r^* is then

$$\begin{aligned} v_i(1 - P(r^*)) + s_i(\sigma)oP(r^*) + t_i((r^*_i, r^*_{-i})) &= v_i + (s_i(\sigma)o - v_i)P(r^*) + t_i((r^*_i, r^*_{-i})) \\ &= v_i + t_i((r^*_i, r^*_{-i})). \end{aligned} \tag{7}$$

If $(0, r^*_{-i})$ is not an equilibrium report profile as well, then (7) must be strictly larger than the utility of seller i under report profile $(0, r^*_{-i})$, which is

$$v_i(1 - P(r)) + s_i(\sigma)oP((0, r^*_{-i})) + t_i((0, r^*_{-i})) = v_i + t_i((0, r^*_{-i})) \geq 0,$$

where the inequality follows from the third condition of Definition 7. It follows that we must have

$$t_i((r^*_i, r^*_{-i})) > t_i((0, r^*_{-i})) \geq 0 \tag{8}$$

for any seller i for whom

1. $s_i(\sigma)o - v_i = 0$ and
2. $(0, r^*_{-i})$ is not an equilibrium report profile.

Now, we suppose that $s_j(\sigma)o - v_j = 0$ for *all* sellers j and consider some equilibrium report profile r^*. Let $\check{I}$ be a set of sellers i such that $(r^*_{\check{I}}, 0)$ is an equilibrium report profile but $(r^*_{\check{I}\setminus i}, 0)$ is *not* an equilibrium report profile for any $i \in \check{I}$. The preceding observations and the third condition of Definition 7 show that

$$\sum_{j \notin \check{I}} t_j((r^*_{\check{I}}, 0)) + \sum_{i \in \check{I}} t_i((r^*_{\check{I}}, 0)) > \sum_{j \notin \check{I}} t_j((r^*_{\check{I}}, 0)) \geq 0. \tag{9}$$

But (9) contradicts the fact that $\mathfrak{M}$ is self-financing unless $\check{I}$ is empty. Thus, we see that $r^* = 0$ must be an equilibrium report profile in the case that $s_j(\sigma)o - v_j = 0$ for all sellers j.

Now, our assumption ruling out correlation of equilibrium reports r^*_i with other sellers' valuations v_{-i} implies that the set of seller i's equilibrium reports r^*_i does not depend on v_{-i}. Hence, as $r^*_i = 0$ is an equilibrium report for seller i in the case that $s_j(\sigma)o - v_j = 0$ for all sellers j, it must also be an equilibrium report for seller i in general; this proves the claim. □

The preceding claim shows that some report profile of the form $(0, r^*_{-i})$ arises in equilibrium when $s_i(\sigma)o - v_i = 0$. In that case, the seller efficiency of $\mathscr{M}$ implies that the decision $P((0, r^*_{-i}))$ maximizes $\sum_j (s_j(\sigma)o - v_j) = \sum_{j \neq i}(s_j(\sigma)o - v_j)$. Our assumption ruling out correlation of equilibrium reports r^*_j with other sellers' valuations v_{-j} implies that this is still true even if $r_i = 0$ is an out-of-equilibrium report, as it would be an equilibrium report in the case that $s_i(\sigma)o - v_i = 0$. Thus, we see that whenever seller i reports $r_i = 0$, the decision $P((0, r^*_{-i}))$ maximizes $\sum_{j \neq i}(s_j(\sigma)o - v_j)$.

Now, as $\tilde{\mathscr{M}}$ is seller-efficient, we know that $\tilde{P}$ maximizes $\sum_{j \neq i}(s_j(\sigma)o - v_j)$ under equilibrium reporting $\tilde{r}^*_{-i}$. Thus, we see that $P((0, r^*_{-i})) = \tilde{P}(\tilde{r}^*_{-i})$; this observation, along with the second condition of Definition 7, shows that $\mathfrak{M}$ guarantees sellers their fair shares.

Finally, as Concordance procedures are seller-efficient, the rest of Theorem 3 follows from Lemma 2.

Proof of Proposition 2

The main result of Green and Laffont (1977) shows that only Groves mechanisms can be seller-efficient in dominant strategy equilibrium. For any such mechanism $\mathscr{M}$, we may normalize the allocation rule P such that $P(r) = 1_{R \geq 0}$; we assume this in the sequel. Now, if $\mathfrak{M} = \mathfrak{M}(\mathscr{M})$ is an approximately individually rational procedure associated to a (normalized) Groves mechanism $\mathscr{M}$, then sellers report $r^*_i = s_i(\hat{\sigma})o - v_i$ in dominant-strategy equilibrium, and the transfer function $t_i(r)$ of $\mathscr{M}$ can be decomposed in the form

$$t_i(r) = -1_{R_{-i} \cdot R < 0}|R_{-i}| + \hat{h}_i(r^*_{-i}), \tag{10}$$

for some $\hat{h}_i(r^*_{-i})$ depending only on $(o, \hat{\sigma})$ and r_{-i}.

Now, as $\mathfrak{M}$ guarantees sellers their fair shares, there must be some $r_i \in \mathcal{R}$ such that

1. $P((r_i, r^*_{-i})) = \tilde{P}(\tilde{r}^*_{-i})$, where $\tilde{P}$ is the decision rule of $\tilde{\mathscr{M}} = \mathscr{M}_{N-1}((1 - s_i(\hat{\sigma}))o, \tilde{\sigma})$, $\tilde{\sigma}$ is chosen so that the share of each seller $j \neq i$ under $\tilde{\mathscr{M}}$ is $\frac{s_j(\hat{\sigma})}{1 - s_i(\hat{\sigma})}$, and $\tilde{r}^*_{-i}$ is the equilibrium report under $\tilde{\mathscr{M}}$, and

2. $t((r_i, r^*_{-i})) \geq 0$.

As $\tilde{\mathscr{M}}$ is seller-efficient, we have $\tilde{P}(\tilde{r}^*_{-i}) = 1$ if and only if

$$0 \leq \sum_{j \neq i}(s_j(\hat{\sigma})o - v_j) = \sum_{j \neq i} r^*_j = R^*_{-i}.$$

Thus, we see that

$$1_{R^*_{-i} \geq 0} = \tilde{P}(\tilde{r}^*_{-i}) = P((r_i, r^*_{-i})) = 1_{R \geq 0},$$

where here $R = R^*_{-i} + r_i$. Hence, $R^*_{-i} \cdot R \geq 0$. From the decomposition (10) and the second condition on r_i, we then have

$$0 \leq t_i((r_i, r^*_{-i})) = -1_{R^*_{-i} \cdot R < 0}|R^*_{-i}| + \hat{h}_i(r^*_{-i}) = \hat{h}_i(r^*_{-i}) \quad \text{(for each seller } i\text{).} \tag{11}$$

Now, we suppose that $\hat{h}_j(r^*_{-j}) > 0$ for some r^*_{-j} and let κ be a constant large enough that $\operatorname{sign}(\kappa R^*_{-j} - r_i) = \operatorname{sign}(R^*_{-j})$ for all $i \neq j$. We take $v_j = s_j(\hat{\sigma})o - \kappa R^*_{-j}$, so that $r^*_j = \kappa R^*_{-j}$ and note that with this choice $R^* = r^*_j + R^*_{-j} = (1 + \kappa)R^*_{-j}$, so that $\operatorname{sign}(R^*) = \operatorname{sign}(R^*_{-j})$ and

$$t_j(r^*) = -1_{R^*_{-j} \cdot R^* < 0}|R^*_{-j}| + \hat{h}_j(r^*_{-j}) = \hat{h}_j(r^*_{-j}) > 0. \tag{12}$$

Meanwhile, for each $i \neq j$, we have

$$R^*_{-i} = R^* - r^*_i = (r^*_j + R^*_{-j}) - r^*_i = \kappa R^*_{-j} + R^*_{-j} - r^*_i = (\kappa R^*_{-j} - r^*_i) + R^*_{-j};$$

hence, $\text{sign}(R^*_{-i}) = \text{sign}(R^*_{-j}) = \text{sign}(R^*)$ by our choice of κ. It thus follows from (11) that

$$t_i(r^*) = -1_{R^*_{-i} \cdot R^* < 0} |R^*_{-i}| + \hat{h}_i(r^*_{-i}) = \hat{h}_i(r^*_{-i}) \geq 0. \tag{13}$$

We then have

$$\sum_i t_i(r^*) = t_j(r^*) + \sum_{i \neq j} t_i(r^*) = \hat{h}_j(r^*_{-j}) + \sum_{i \neq j} \hat{h}_i(r^*_{-i}) > 0$$

by (12) and (13), which contradicts the fact that $\mathscr{M}$ is self-financing. Thus, we cannot have $\hat{h}_j(r^*_{-j}) > 0$ for any j; whence we see that $\hat{h}_j(r^*_{-j}) \leq 0$ for all j. Combining this observation with (11) shows that in fact $\hat{h}_i(r^*_{-i}) = 0$ for all i.

As our assumptions on the distribution of values implies that each possible report profile arises in equilibrium, we thus see that in fact $\hat{h}_i = 0$ for all i; this proves the result.

Proof of Proposition 4

The main result of Williams (1999) implies that a Bayes-Nash implementable, seller-efficient mechanism must be interim-equivalent to a Groves mechanism. This observation narrows the space of mechanisms so that for each i,

$$t_i(r) - s_i o P(r) = -\text{EE}_i(r_i) + \hat{h}_i(r^*_{-i}),$$

for some $\hat{h}_i(r^*_{-i})$ depending only on $(o, \hat{\sigma})$ and r_{-i}. Budget-balance then implies that $\sum_i \text{EE}_i(r_i) = \sum_i \hat{h}_i(r^*_{-i})$; hence, the only flexibility in a budget-balanced, Bayes-Nash implementable, seller-efficient mechanism is in the form of the tax refunds $\hat{h}_i(r^*_{-i})$. The requirement that $\frac{t_i}{t_j} = \frac{s_i(\hat{\sigma})}{s_j(\hat{\sigma})}$ when $r_i = r_j = 0$ pins down these refunds as exactly those of EEC.