Market Design Approaches to Inequality II: Design of Affirmative Action Mechanisms

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Overview

Yesterday

- Balancing Fairness, Efficiency, and Incentives

Today

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

The Setting

- Centralized assignment of K-12 public school seats.
- Students (i.e. their parents) are (potentially) strategic agents.
- School seats are "goods"; students have unit demand.
- Students' priorities at schools are **exogenous**.

Basic Theory (Abdulkadiroğlu–Sönmez, 2003)

- $I \sim$ set of students
- $C \sim$ set of schools

- $P^i \sim$ preference ranking of $i \in I$ over schools (and $\emptyset$)

- $\Pi^c \sim$ priority ranking of $c \in C$ over students
- $q_c \sim$ total capacity of $c \in C$

- A **match** μ specifies an assignment of students to schools.
  (must respect capacities – $|\mu(c)| \leq q_c$)

- A **mechanism** φ assigns a match, given submitted preferences.

Basic Design Goals

- **Individual Rationality** ($\sim$ participation)
  - No student wants to drop out (i.e. $\mu(i) P^i \emptyset$).
- **Elimination of Justified Envy** ($\sim$ stability)
  - If i envies j, then j has higher priority than i at $\mu(j)$ (i.e. $\mu(j) P^i \mu(i) \implies j \Pi^{\mu(j)} i$).
- **Strategy-proofness**
  - Truthfulness is dominant (i.e. $\varphi(P^i, P^{-i}) P^i \varphi(\bar{P}^i, P^{-i})$).
- **Pareto Efficiency**
- **Respect of (unambiguous) Improvements in Priority**

"No choice in school choice?" (Calsamiglia–Miralles, 2012)

Definition

A mechanism **does not respect the spirit of school choice** if it always assigns students to their "neighborhood schools."

Bad News

- In a large market with binary, "neighborhood" priority and agreement as to the worst school, neither SOSM nor Boston respects the spirit of school choice.
- Natural "fixes" involve favoring already-advantaged students.

Slightly Better News

- TTC does respect the spirit of school choice (but does not help students who live in the bad neighborhood).

Today

Use insights from matching theory to inform the design of affirmative action mechanisms.

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Backdrop

Affirmative Action refers to positive steps aimed at increasing the inclusion of historically excluded groups in employment, education and business. Such steps are not designed to offer preferential treatment to, or exclude from participation, any group. To the contrary, Affirmative Action policies are intended to promote access for the traditionally underrepresented through heightened outreach and efforts at inclusion.

(American Association for Affirmative Action)

Backdrop

"Affirmative action" means positive steps taken to increase the representation of women and minorities in areas of employment, education, and business from which they have been historically excluded. When those steps involve [. . .] selection on the basis of race, gender, or ethnicity [. . .] affirmative action generates intense controversy.

(Stanford Encyclopedia of Philosophy)

Backdrop

Forty years ago, as the United States experienced the civil rights movement [. . .]. After a full generation [. . .] a plethora of government-enforced diversity policies have marginalized many white workers.

(Sen. James Webb, 2010)

The Civil Rights Act of 1964 gave the federal government unprecedented power over the hiring, employee relations, and customer service practices of every business in the country. The result was a massive violation of the rights of private property and contract, which are the bedrocks of free society.

(Rep. Ron Paul, 2004)

Background

While there is (heated) disagreement about the value of affirmative action; there is little disagreement about what affirmative action actually *does*.

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But. . .

Popular “majority quota”-based affirmative action policies can hurt *every minority student*.

Extended Model (Kojima, 2012)

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  (must respect capacities – $|\mu(c)| \leq q_c$ –
  and quotas – $|\mu(c) \cap I_{\mathrm{M}}| \leq q_c^{\mathrm{M}}$)

Extended Model (Kojima, 2012)

Definition

A match is **stable** (in the presence of quotas) if

1. it is **individually rational** – $\mu(i) P^i \emptyset$ for all $i \in I$ – and
2. it is **unblocked** – if $c P^i \mu(i)$, then either
   - $|\mu(c)| = q_c$ and $j \Pi^c i$ for all $j \in \mu(c)$, or
   - $\mathrm{t}(i) = \mathrm{M}$, c's majority quota is met (i.e. $|\mu(c) \cap I_{\mathrm{M}}| = q_c^{\mathrm{M}}$), and all $j \in (\mu(c) \cap I_{\mathrm{M}})$ have higher priority at c than i.

(This looks really complicated. . .)

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Definition

A mechanism is **stable** if it always selects stable outcomes.

Affirmative Action with Majority Quotas (Kojima, 2012)

Definition

A mechanism **respects the spirit of quota-based affirmative action** if when majority quotas are decreased ($q_c^{\mathrm{M}} \rightarrow \tilde{q}_c^{\mathrm{M}} < q_c^{\mathrm{M}}$), minority outcomes improve.

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Bad News

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$$\Pi^{c_1} : i_1^{\mathrm{M}} \succ i_2^{\mathrm{M}} \succ i^{\mathrm{m}} \succ \emptyset$$
$$\Pi^{c_2} : i_2^{\mathrm{M}} \succ i^{\mathrm{m}} \succ i_1^{\mathrm{M}} \succ \emptyset$$

$$q_{c_1} = 2; \quad q_{c_1}^{\mathrm{M}} = 2$$
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Bad News

- No stable mechanism respects the spirit of quota-based affirmative action. (The quota outcome can be Pareto inferior!)
- Quotas $\sim$ unpredictable. (They *can* cause Pareto improvement.)
- Similar results for {"priority-based" affirmative action, TTC}.

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(But wait. . .)

Re-examining the Example

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Key Observation (I)

The reason that a quota for majority students can have adverse effects on minority students is simple. Consider a situation in which a school c is mostly desired by majorities. Then having a majority quota for c decreases the number of majority students that can be assigned to c even if there are empty seats. This, in turn, increases the competition for other schools and thus can even make the minority students worse off.

(Hafalir–Yenmez–Yildirim, forth.)

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Key Observation (II)

The number of minority students preferring a school to another is not known a priori *by the policymakers. Even most intelligent guesses of quota levels will be prone to small deviations in minority students' realized desire to attend a particular school, which might cascade inefficiencies throughout the system. [...] Moreover, these quotas are usually set by third parties such as courts or school districts, which means that they cannot be readjusted easily if schools have empty seats.*

(Hafalir–Yenmez–Yildirim, forth.)

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Extended Model (Hafalir–Yenmez–Yildirim, forth.)

- I ~ set of students (each $i \in I$ has type M or m)
- C ~ set of schools

- P^i ~ preference ranking of $i \in I$ over schools (and $\emptyset$)

- Π^c ~ priority ranking of $c \in C$ over students
- q_c ~ total capacity of $c \in C$; r_c^{m} ~ minority reserve of $c \in C$

- A **match** μ specifies an assignment of students to schools.

  (must respect capacities – $|\mu(c)| \leq q_c$ – and reserves)

Extended Model (Hafalir–Yenmez–Yildirim, forth.)

Definition

A match is **stable** (in the presence of reserves) if

1. it is **individually rational** and
2. it is **unblocked** – if $cP^i\mu(i)$, then $|\mu(c)| = q_c$ and either
   - $\mathrm{t}(i) = \mathrm{m}$ and all $j \in \mu(c)$ have higher priority than i,
   - $\mathrm{t}(i) = \mathrm{M}$, c's reserved slots are full, and all $j \in \mu(c)$ have higher priority at c than i, or
   - $\mathrm{t}(i) = \mathrm{M}$, c's reserved slots are not full, and all $j \in (\mu(c) \cap I_{\mathrm{M}})$ have higher priority at c than i.

Minority Reserves "Work" (Hafalir–Yenmez–Yildirim, forth.)

1. For any match μ stable under quotas q^{M}, there exists a match stable under reserves $r^{\mathrm{m}} = q - q^{\mathrm{M}}$ that Pareto improves on μ.
2. Minority students never (Pareto) prefer the SOSM without affirmative action to the SOSM with minority reserves.
3. Under natural conditions, minority students (Pareto) prefer SOSM with reserves to the SOSM without affirmative action.
4. Similar results hold for "TTC with minority reserves."

Simulations Say More (Hafalir–Yenmez–Yildirim, forth.)

1. Reserves improve minority welfare (but can hurt majorities).
2. SOSM with minority reserves “significantly” Pareto dominates SOSM with majority quotas (for all students).
3. Quota-based mechanism outcomes are sensitive to quota size.
4. Students on average prefer TTC to SOSM for all affirmative action policies.

Related Approaches

- **Regional Quotas** (Kamada–Kojima, 2011)
- **“Complex Constraints”** (Westkamp, 2012)
- **Slot-Specific Priorities** (K.–Sönmez, 2012)

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\pause

Last component of the talk—current/ongoing research.

\pause

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(Please mind the notation.)

So far...

Theory $\longrightarrow$ Practice $\longrightarrow$ Evaluation

Meanwhile. . .

Theory $\longrightarrow$ Practice $\longrightarrow$ Evaluation

Now. . .

Theory $\longrightarrow$ Practice $\longrightarrow$ Evaluation

Now. . .

Theory $\longleftarrow$ Evaluation

Socioeconomic Affirmative Action in Chicago

`The Setting:`

- Students $\leftrightarrow$ Elite Public High Schools

`The Problem:`

- 15% of slots are reserved for each class $(t \in \{4, 3, 2, 1\})$.

$\Rightarrow$ Priorities vary across slots.

`Ad Hoc` `Solution:`

- Divide each school into (five) sub-schools;
- Run Abdulkadiroğlu–Sönmez (2003) SOSM, filling "open" sub-schools before "reserved" ones.

Thought Experiment

Chicago School Choice:
Test scores $\Rightarrow$ global priority π; some slots have minority reserves.

$$S^{\circ} : \{4, 3, 2, 1\}$$
$$S^{4} : 4 \succ \{3, 2, 1\}$$
$$S^{3} : 3 \succ \{4, 2, 1\}$$
$$S^{2} : 2 \succ \{4, 3, 1\}$$
$$S^{1} : 1 \succ \{4, 3, 2\}$$

Facts

- Minorities (e.g., Tier 1) have systematically low test scores.
- 16,372 students compete for 4,270 elite high school slots.

Thought Experiment

Chicago School Choice:
Test scores $\Rightarrow$ global priority π; some slots have minority reserves.

$S^{o} : \{4, 3, 2, 1\}$
$S^4 : 4 \succ \{3, 2, 1\}$
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Facts

- Minorities (e.g., Tier 1) have systematically low test scores.
- 16,372 students compete for 4,270 elite high school slots.

Impact of Chicago Slot-Precedence Order (I)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

Current Mechanism:

$S^{\circ} : \{4, 3, 2, 1\}$

$S^4 : 4 \succ \{3, 2, 1\}$

$S^3 : 3 \succ \{4, 2, 1\}$

$S^2 : 2 \succ \{4, 3, 1\}$

$S^1 : 1 \succ \{4, 3, 2\}$

Counterfactual:

$S^4 : 4 \succ \{3, 2, 1\}$

$S^3 : 3 \succ \{4, 2, 1\}$

$S^2 : 2 \succ \{4, 3, 1\}$

$S^1 : 1 \succ \{4, 3, 2\}$

$S^{\circ} : \{4, 3, 2, 1\}$

Total changeover: 766 slots $\approx$ 18%

Impact of Chicago Slot-Precedence Order (II)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Current Mechanism (Open Slots First)				Effect of Switching (to fill Open Slots Last)			
	Tier 4	Tier 3	Tier 2	Tier 1	Tier 4	Tier 3	Tier 2	Tier 1
1	105	71	47	43	30	−18	−8	−4
2	95	114	70	60	0	24	−14	−10
3	87	78	86	73	−36	16	35	−15
4	106	93	80	68	−21	9	28	−16
5	210	100	78	78	20	−2	−9	−9
6	121	69	45	49	25	−15	−3	−7
7	655	412	291	272	29	37	−38	−28
8	90	47	36	36	3	7	−5	−5
9	92	129	90	94	−27	5	19	3
TOT	1561	1113	823	773	23	63	5	−91

Impact of Chicago Slot-Precedence Order (III)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Counterfactual Mechanism (Open Slots Last)				
	Tier 4	Tier 3	Tier 2	Tier 1	Reserved
1	135	53	39	39	39
2	95	138	56	50	50
3	51	94	121	58	48
4	85	102	108	52	52
5	230	98	69	69	69
6	146	54	42	42	42
7	684	449	253	244	244
8	93	54	31	31	31
9	65	134	109	97	60

“Reserves” convereted into “Quotas”

Impact of Chicago Slot-Precedence Order (IV)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Tier 4	Tier 3	Tier 2	Tier 1
Prefer Actual	62	50	108	175
Indifferent	3748	4287	4092	3474
Prefer Counterfactual	225	108	43	0

Tier 1 unambiguously prefers the current mechanism.

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The Setting:

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The Problem:

- 15% of slots are reserved for each class ($t \in \{4, 3, 2, 1\}$).

$\Rightarrow$ Priorities vary across slots.

Ad Hoc Solution:

- Divide each school into (five) sub-schools;
- Run Abdulkadiroğlu–Sönmez (2003) SOSM, filling "open" sub-schools before "reserved" ones.

Neighborhood Priority in Boston

The Setting:

- Students $\leftrightarrow$ K–12 Public Schools

The Problem:

- 50% of slots give "walk-zone priority."

$\Rightarrow$ Priorities vary across slots.

Ad Hoc Solution:

- Divide each school into (two) sub-schools;
- Run Abdulkadiroğlu–Sönmez (2003) SOSM, filling "walk-zone" sub-schools before "open" ones.

Side-by-Side Comparison

Chicago	Boston
Open	Walk-Zone
Reserved	Open

Cadet–Branch Matching

The Setting:

- US Military Cadets $\leftrightarrow$ Branches of Service

The Problem:

- To increase retention, allow "bidding" for bottom 25% of slots.

$\Rightarrow$ Priorities vary across slots.

Ad Hoc Solution:

- Divide each branch into (two) sub-branches;
- Run Hatfield–Kojima (2010) cumulative offer mechanism, filling "regular" sub-branches before "branch-of-choice" sub-branches.

Matching with Slot-Specific Priorities (K.–Sönmez, 2012)

- Affirmative Action in Chicago
- Neighborhood Priority in Boston
- Cadet–Branch Matching

`Problem:`

Priorities vary across slots.

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- Affirmative Action in Chicago
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`Problem:`

Priorities vary across slots.

Now

A general and unified framework for these applications in which:

- Key substitutability conditions do not hold.
- Agent-optimal stable outcomes may not exist.
- The cumulative offer mechanism is nevertheless *stable*, *strategy-proof*, and *improvement-respecting*.

Model (Informal)

- *Agents* have preferences over *contracts* with *branches*.

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  - has its own priority order, and
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Model (Informal)

- *Agents* have preferences over *contracts* with *branches*.
- Each branch has *slots* which can be assigned contracts.
- Each slot
    - has its own priority order, and
    - can hold at most one contract.
- Slots are filled sequentially, according to *precedence order*.

Model (Formal)

- Set I of agents; set B of branches.
- Set $X \subseteq I \times B \times T$ of contracts.

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  - Choice C^i defined by maximization.

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- Order of slot-precedence $\rhd^b$ over S^b.

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- Set S^b of slots at branch b.
- Order of slot-precedence $\rhd^b$ over S^b.

- $\Pi^s \sim$ priorities of $s \in S^b$ over $X_b \equiv \{x \in X : \mathrm{b}(x) = b\}$.
  - Choice C^b defined by $\rhd^b$-sequential maximization.

$\rhd^b$-sequential Maximization: An Example

$$\Pi^{s_b^1} : y \succ x \succ z \succ \emptyset$$

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$$C^b(\{x, z\}) = \{x\}$$

$\rhd^b$-sequential Maximization: An Example

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$$\Pi^{s_b^2} : x \succ \emptyset$$

$$C^b(\{x, z\}) = \{x\} \qquad C^b(\{x, y, z\}) = \{y, x\}$$

Embedded Models

- "Classical" School Choice
  - Balinski–Sönmez (1999); Abdulkadiroğlu–Sönmez (2003) ...
  - Abdulkadiroğlu–Pathak–Roth (2005; 2009); Abdulkadiroğlu–Pathak–Roth–Sönmez (2005) ...
- Affirmative Action in School Choice
  - Quotas $\sim$ Abdulkadiroğlu–Sönmez (2003); Abdulkadiroğlu (2005); Kojima (2012) ...
  - Reserves $\sim$ Hafalir–Yenmez–Yildirim (forth.)
- Cadet–Branch Matching
  - Sönmez–Switzer (2011); Sönmez (2011)

Solution Concept

Definition

An outcome $Y \subseteq X$ is **stable** if it is

1. **Individually Rational**:
   - $C^i(Y) = Y_i$ for all $i \in I$;
   - $C^b(Y) = Y_b$ for all $b \in B$.
2. **Unblocked**: There does not exist a nonempty **blocking set** $Z \not\subseteq Y$ such that
   - $Z_i \subseteq C^i(Y \cup Z)$ for all $i \in \mathrm{i}(Z)$;
   - $Z_b \subseteq C^b(Y \cup Z)$ for all $b \in \mathrm{b}(Z)$.

The Cumulative Offer Process (I)

Step 1

1. One agent "proposes" her first-choice contract, x.
2. $A_2^{\mathrm{b}(x)} = \{x\}$; $A_2^b \equiv \varnothing$ for $b \neq \mathrm{b}(x)$.
3. Each branch b "holds" $C^b(A_2^b)$.
   - I.e. if x is acceptable to some $s \in S^{\mathrm{b}(x)}$, then $\mathrm{b}(x)$ holds x.

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    - I.e. if x is acceptable to some $s \in S^{\mathrm{b}(x)}$, then $\mathrm{b}(x)$ holds x.

Step $\ell \geq 2$

1. Some agent for whom no contract is held proposes a contract y which has not yet been rejected.
2. $A_{\ell+1}^{\mathrm{b}(y)} = A_\ell^{\mathrm{b}(y)} \cup \{y\}$; $A_{\ell+1}^b \equiv A_\ell^b$ for $b \neq \mathrm{b}(y)$.
3. Each branch b holds $C^b(A_{\ell+1}^b)$.

The Cumulative Offer Process (II)

Definition

The **cumulative offer mechanism** Φ_Π imposes the outcome of the cumulative offer process under priorities Π and submitted preferences.

Central Result

Theorem

The cumulative offer mechanism Φ_{Π} is stable and strategy-proof.

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(Observation: This *looks* like a standard result.)

Opposition of *Agents'* Interests

Example

- $I = \{i, j, k\}$; $B = \{b\}$; $S^b = \{s_b^1, s_b^2\}$; $X = \{i_1, i_2, j_1, j_2, k_1, k_2\}$.

$$P^i : i_1 \succ i_2 \succ \emptyset$$
$$P^j : j_1 \succ j_2 \succ \emptyset$$
$$P^k : k_1 \succ k_2 \succ \emptyset$$

$$\Pi^{s_b^1} : i_2 \succ j_2 \succ k_2 \succ i_1 \succ j_1 \succ k_1 \succ \emptyset$$
$$\Pi^{s_b^2} : i_1 \succ i_2 \succ j_1 \succ j_2 \succ k_1 \succ k_2 \succ \emptyset$$

- Stable outcomes: $Y \equiv \{j_2, i_1\}$ and $Y' \equiv \{i_2, j_1\}$.
- i prefers Y; j prefers Y'.

Our Approach

Theorem

The cumulative offer mechanism Φ_{Π} is stable and strategy-proof.

Proof Strategy

1. Construct associated *one-to-one* agent–slot matching market.
   - "Slot-stable" outcomes induce stable outcomes (by projection).
2. Show that the cumulative offer process corresponds to the agent-optimal slot-stable mechanism in the agent–slot market.
   1. Slots' contracts improve during the cumulative offer process.
   2. The cumulative offer process outcome $\sim$ slot-stable...
   3. ...and agent-optimal in the agent–slot market.

Completing the Proof

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This implies that the proposal order does not affect the outcome.

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Corollary

This implies that the proposal order does not affect the outcome.

"Corollary" (aka Central Theorem)

The cumulative offer mechanism Φ_{Π} is stable and strategy-proof.

Additional Properties of Φ_{Π} (I)

Recall: Agent-optimal stable outcomes may not exist.

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Proof

- More generally: No stable outcome Z can Pareto dominate the cumulative offer process outcome Y.
- To see this, we consider alternative proposal order in which
  1. all agents in $\mathrm{i}(Z)$ who wish to propose contracts weakly preferred to those in Z have the opportunity to propose contracts before any agents in $I \setminus \mathrm{i}(Z)$ do, and
  2. all agents in $\mathrm{i}(Z)$ who wish to propose contracts *not* weakly preferred to those in Z are not allowed to propose unless no agents in $I \setminus \mathrm{i}(Z)$ wish to propose contracts.

Additional Properties of Φ_{Π} (II)

Definition

$\bar{\Pi}$ is an unambiguous improvement over priority profile Π for $i \in I$ if $\bar{\Pi}$ is obtained from Π by raising the priorities of some of i's contracts (at some slots).

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Φ_Π *respects unambiguous improvements in agent priority.*

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Definition

$\bar{\Pi}$ is an unambiguous improvement over priority profile Π for $i \in I$ if $\bar{\Pi}$ is obtained from Π by raising the priorities of some of i's contracts (at some slots).

Theorem

Φ_Π *respects unambiguous improvements in agent priority.*

Proof

- Once again, we consider an alternative proposal order: i proposes contracts only when no other agent is able to propose.
- Under this order i is always the last agent to propose.
- Last contract proposed facing $\bar{\Pi}$ also proposed facing Π.

Summary of Results

Theorem

The cumulative offer mechanism Φ_{Π}

1. *is stable and strategy-proof,*
2. *is independent of proposal order,*
3. *selects agent-optimal stable outcomes when they exist, and*
4. *respects unambiguous improvements in agent priority.*

Summary of Results

Theorem

The cumulative offer mechanism Φ_Π

1. *is stable and strategy-proof,*
2. *is independent of proposal order,*
3. *selects agent-optimal stable outcomes when they exist, and*
4. *respects unambiguous improvements in agent priority.*

Further Applications

- We can re-derive prior results on quotas vs. reserves.
- Unbiased mechanism for Chicago school choice program.
- Impact of slot-precedence in cadet–branch matching.

Thought Experiment: Revisited

Chicago School Choice:

Test scores $\Rightarrow$ global priority π; some slots have minority reserves.

$S^{\circ} : \{4, 3, 2, 1\}$

$S^4 : 4 \succ \{3, 2, 1\}$

$S^3 : 3 \succ \{4, 2, 1\}$

$S^2 : 2 \succ \{4, 3, 1\}$

$S^1 : 1 \succ \{4, 3, 2\}$

$S^4 : 4 \succ \{3, 2, 1\}$

$S^3 : 3 \succ \{4, 2, 1\}$

$S^2 : 2 \succ \{4, 3, 1\}$

$S^1 : 1 \succ \{4, 3, 2\}$

$S^{\circ} : \{4, 3, 2, 1\}$

Facts

- Minorities (e.g., tier 1) have systematically low test scores.
- 16,372 students compete for 4,270 elite high school slots.

Thought Experiment: Revisited

Theory gives no clear answer:

- $I = \{\mathrm{M}, \mathrm{m}_1, \mathrm{m}_2, \mathrm{m}_3\}$; $B = \{b, b'\}$.

$P^{\mathrm{M}} : b' \succ b \succ \emptyset \qquad P^{\mathrm{m}_1} : b' \succ b \succ \emptyset \qquad P^{\mathrm{m}_2} : b' \succ \emptyset \qquad P^{\mathrm{m}_3} : b \succ \emptyset.$

Test scores $\Rightarrow$ global priority order

$$\pi : \mathrm{m}_1 \succ \mathrm{M} \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \emptyset.$$

$$\Pi^{s_b^{\mathrm{o}}} : \mathrm{m}_1 \succ \mathrm{M} \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \emptyset$$

Option 1:

$\Pi^{s_{b'}^{\mathrm{m}}} : \mathrm{m}_1 \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \mathrm{M} \succ \emptyset$

$\Pi^{s_{b'}^{\mathrm{o}}} : \mathrm{m}_1 \succ \mathrm{M} \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \emptyset$

Option 2:

$\Pi^{s_{b'}^{\mathrm{o}}} : \mathrm{m}_1 \succ \mathrm{M} \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \emptyset$

$\Pi^{s_{b'}^{\mathrm{m}}} : \mathrm{m}_1 \succ \mathrm{m}_2 \succ \mathrm{m}_3 \succ \mathrm{M} \succ \emptyset$

Impact of Chicago Slot-Precedence Order (I)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

Current Mechanism:

$$S^{o} : \{4, 3, 2, 1\}$$
$$S^4 : 4 \succ \{3, 2, 1\}$$
$$S^3 : 3 \succ \{4, 2, 1\}$$
$$S^2 : 2 \succ \{4, 3, 1\}$$
$$S^1 : 1 \succ \{4, 3, 2\}$$

Counterfactual:

$$S^4 : 4 \succ \{3, 2, 1\}$$
$$S^3 : 3 \succ \{4, 2, 1\}$$
$$S^2 : 2 \succ \{4, 3, 1\}$$
$$S^1 : 1 \succ \{4, 3, 2\}$$
$$S^{o} : \{4, 3, 2, 1\}$$

Total changeover: 766 slots $\approx$ 18%

Impact of Chicago Slot-Precedence Order (II)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Current Mechanism (Open Slots First)				Effect of Switching (to fill Open Slots Last)			
	Tier 4	Tier 3	Tier 2	Tier 1	Tier 4	Tier 3	Tier 2	Tier 1
1	105	71	47	43	30	−18	−8	−4
2	95	114	70	60	0	24	−14	−10
3	87	78	86	73	−36	16	35	−15
4	106	93	80	68	−21	9	28	−16
5	210	100	78	78	20	−2	−9	−9
6	121	69	45	49	25	−15	−3	−7
7	655	412	291	272	29	37	−38	−28
8	90	47	36	36	3	7	−5	−5
9	92	129	90	94	−27	5	19	3
TOT	1561	1113	823	773	23	63	5	−91

Impact of Chicago Slot-Precedence Order (III)

Simulation confirms our intuition:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Counterfactual Mechanism (Open Slots Last)				
	Tier 4	Tier 3	Tier 2	Tier 1	Reserved
1	135	53	39	39	39
2	95	138	56	50	50
3	51	94	121	58	48
4	85	102	108	52	52
5	230	98	69	69	69
6	146	54	42	42	42
7	684	449	253	244	244
8	93	54	31	31	31
9	65	134	109	97	60

“Reserves” convereted into “Quotas”

An Engineering Problem

What would an "unbiased" mechanism for the Chicago school choice program look like?

Chicago with an Unbiased Mechanism

Simulation yields an (initially) surprising result:
Treating the Chicago school choice mechanism as (fully) strategy-proof, we can compute counterfactual assignments.

- Recall: 16,372 students; 4,270 elite high school slots.

	Counterfactual Mechanism (Open Slots Last)				Effect of Switching (to Unbiased Mechanism)			
	Tier 4	Tier 3	Tier 2	Tier 1	Tier 4	Tier 3	Tier 2	Tier 1
1	135	53	39	39	4	−2	−1	−1
2	95	138	56	50	2	−2	0	0
3	51	94	121	58	−5	4	3	−2
4	85	102	108	52	−2	1	1	0
5	230	98	69	69	−1	2	−1	0
6	146	54	42	42	1	−1	0	0
7	684	449	253	244	−4	3	1	0
8	93	54	31	31	0	0	0	0
9	65	134	109	97	−3	0	0	3
TOT	1584	1176	828	682	−8	5	3	0

Overview

Yesterday

- Balancing Fairness, Efficiency, and Incentives

Today

- Quota-Based Mechanisms
- Minority Reserves
- Slot-Specific Priorities

What is Market Design?

Application of economic principles and game theory to the design (or re-design) of market institutions.

Why do we care?

Balancing fairness, efficiency, and incentives can be *hard*.

How is market design related to inequality?

Effective design can reduce frictions and help ensure equal access to the benefits of the market.

Where is market design effective?

Many of our successes thus far have been in self-contained markets, with institution-driven market failures.

Who should practice market design?

Maybe you!

Who should practice market design?