Dynamic Position Auctions with Consumer Search

Scott Duke Kominers

Harvard University

Algorithmic Aspects in Information and Management
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What are position auctions?

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- Position auctions are used to allocate sponsored search links to advertisers!

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- The mechanisms used are relatively new
- Welfare implications not well-understood

Previous Position Auction Models

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Exogenous
Click-through Rates

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    - Advertisers play a “best-response” strategy

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- Dynamic setting
    - Sequential rounds
    - Synchronous updating
    - Advertisers play a “best-response” strategy
    - Consumers ignorant of dynamics

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$$G(\bar{q}_j) \cdot (1 - q_\pi) \cdot (q_\pi - b_{\pi_{j+1}}) = G(\bar{q}_{j-1}) \cdot (q_\pi - b_\pi)$$

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  - Athey and Ellison (2008) Envy-Free Equilibrium

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Restricted Balanced Bidding

- Given the bids of the other advertisers, advertiser π
  - targets the position j among the positions below the current position which maximizes utility
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- The dynamic model is “well-approximated” by the static model.

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- $\gamma^*(q) = (1 - q) \max_{j>0} \left(\frac{G(\bar{q}_j)}{G(\bar{q}_{j-1})} \right)$
- $\gamma^{**} = \max_{1 \leq \pi \leq N} \gamma^*(q_\pi)$

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Lemma

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At every round
$t > t_1 = 2 + \log_{\gamma^{**}}((1 - \gamma^{**})(q_M - q_{M+1})/q_{M+1})$*:*

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$$\begin{cases} b_\pi > q_{M+1} & \pi < M + 1, \\ b_\pi = q_\pi & \pi \geq M + 1. \end{cases}$$

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- Within t_1 rounds, the $N - M$ lowest-quality advertisers "drop out" of contention.

Results

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- By the Lemma

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  - Next round, all advertisers in $\pi(P)$ repeat their bids.
    - If $\pi(P) = \{1, \ldots, M\}$, then we are done.

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  - Look at the advertiser $\pi \notin \pi(P)$ with the lowest bid.

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- Set of stable positions: $P = \{p + 1, \ldots, M\}$
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  - $\Rightarrow P' = P \cup \{p\}$ is stable

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Convergence of the M Positions

- Set of stable positions: $P = \{p + 1, \ldots, M\}$
- Advertiser $\pi \notin \pi(P)$ with the lowest bid
  - Case 2: π targets position $\hat{p} > p$

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    - Depends upon the specific functional form of $\gamma_{\hat{p}}(q)$

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        - Depends upon the specific functional form of $\gamma_{\hat{p}}(q)$
        - (Significant divergence from Cary et al. (2008))

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Convergence of the M Positions

- Set of stable positions: $P = \{p + 1, \ldots, M\}$
- Advertiser $\pi \notin \pi(P)$ with the lowest bid
    - Case 3: π targets position $\hat{p} < p$

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    minimum bid of advertisers not in $\pi(P)$ increases

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Let
$\epsilon = \frac{G(\bar{q}_M)}{2G(\bar{q}_1)}(1-\gamma^{**}) \min_{\phi \neq \phi'} |q_\phi - q_{\phi'}| \left(\prod_{j=1}^{M}(1-q_j)\right).$

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$\epsilon = \frac{G(\bar{q}_M)}{2G(\bar{q}_1)}(1-\gamma^{**}) \min_{\phi \neq \phi'} |q_\phi - q_{\phi'}| \left(\prod_{j=1}^{M}(1-q_j)\right)$.
At most $\log_{1/\gamma^{**}}((q_1 - q_{M+1})/\epsilon)$ *consecutive instances of Case 3 may occur.*

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    after finitely many rounds, Case 1 or 2 must occur

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  - $\Rightarrow$ P remains stable
    minimum bid of advertisers not in $\pi(P)$ increases
    after finitely many rounds, Case 1 or 2 must occur
  - $\Rightarrow$ $\mathbb{QED}$

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- This also yields probability-1 efficient convergence in an asynchronous bidding model.

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- Our method $\approx$ Cary et al. (2008)'s method

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Three key steps:

Three key conditions:

Possible Generalizations

- Our method ≈ Cary et al. (2008)'s method

Three key steps:

1. restriction of the strategy space

Three key conditions:

1. unique envy-free equilibrium

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2. analysis of low-quality advertisers' behaviors

Three key conditions:

1. unique envy-free equilibrium
2. low-quality advertisers drop out efficiently

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- Our method $\approx$ Cary et al. (2008)'s method

Three key steps:

1. restriction of the strategy space
2. analysis of low-quality advertisers' behaviors
3. proof that the M positions stabilize

Three key conditions:

1. unique envy-free equilibrium
2. low-quality advertisers drop out efficiently
3. monotone equilibrium strategy

Conclusion

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Convergence should be demonstrable in dynamic position auction models with sufficiently well-behaved static equilibrium strategies.

Questions?

kominers@fas.harvard.edu