Dynamic Position Auctions with Consumer Search

Scott Duke Kominers

Harvard University

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Road Map

Road Map

- Position Auctions

Road Map

- Position Auctions (without consumer search)

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  - Motivation

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  - Static Model

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- Sponsored search is a multibillion-dollar industry
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- Welfare implications not well-understood

Framework & Conventions

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Edelman, Ostrovsky, and Schwarz (2007) Static Model

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- N advertisers

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  - By convention: $\theta_j = 0 \qquad (j = M + 1, \ldots, N)$

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$$\text{Payoff to } \pi_j\text{: } \theta_j(q_{\pi_j} - b_{\pi_{j+1}})$$

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Equilibria

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$\boxed{\text{Locally Envy-free}}$ Equilibria

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- Envy-free equilibria are *stable*

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- *Do equilibrium results break down once dynamics are introduced?*

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Cary et al. (2008) Dynamic Model

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    - Sequential rounds

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Cary et al. (2008) Dynamic Model

- Extends Edelman, Ostrovsky, and Schwarz (2007)
- Dynamic setting
  - Sequential rounds
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  - Advertisers play a “best-response” strategy

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Balanced Bidding

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- Unique fixed point: Bids follow a recursive formula,

$$b_{\pi_j} = \begin{cases} 2b_{\pi_2} & j = 1, \\ q_{\pi_j} - \frac{\theta_j}{\theta_{j-1}}(q_{\pi_j} - b_{\pi_{j+1}}) & 1 < j \leq M, \\ q_{\pi_j} & M < j \leq N. \end{cases}$$

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Restricted Balanced Bidding

- Given the bids of the other advertisers, advertiser π
  - targets the position j among the positions below the current position which maximizes utility
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- The dynamic model is "well-approximated" by the static model.

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    - Consumer behavior determines click-through rate
    - Consumer- and advertiser-welfare may be aligned
- Implementation of endogenous click-through rate

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$$(1-q_{\pi_1})\cdots(1-q_{\pi_{j-1}})G(\bar{q}_j)$$

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- Dynamic setting
    - Sequential rounds

Framework & Conventions

Our Dynamic Model

- Extends Athey and Ellison (2008)
- Dynamic setting
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Framework & Conventions

Our Dynamic Model

- Extends Athey and Ellison (2008)
- Dynamic setting
  - Sequential rounds
  - Synchronous updating
  - Advertisers play a “best-response” strategy
  - Consumers ignorant of dynamics

Framework & Conventions

Balanced Bidding

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- Given the bids of the other advertisers, advertiser π ...

Framework & Conventions

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- Given the bids of the other advertisers, advertiser π
    - targets the position j which maximizes utility

Framework & Conventions

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- Given the bids of the other advertisers, advertiser π
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  - chooses a bid b_π to satisfy the envy-free condition:

$$G(\bar{q}_j) \cdot (1 - q_\pi) \cdot (q_\pi - b_{\pi_{j+1}}) = G(\bar{q}_{j-1}) \cdot (q_\pi - b_\pi)$$

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  - Athey and Ellison (2008) Envy-Free Equilibrium

Framework & Conventions

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Framework & Conventions

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Framework & Conventions

Restricted Balanced Bidding

- Given the bids of the other advertisers, advertiser π
  - targets the position j $\boxed{\text{among the positions below the current position}}$ which maximizes utility
  - chooses a bid b_π to satisfy the envy-free condition:

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Results

Main Result

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Theorem (Convergence Theorem)

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If all advertisers play the Restricted Balanced Bidding strategy, then their bids converge to the fixed point; this convergence is efficient.

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Theorem (Convergence Theorem)

If all advertisers play the Restricted Balanced Bidding strategy, then their bids converge to the fixed point; this convergence is efficient.

- The dynamic model is "well-approximated" by the static model.

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- $\gamma^*(q) = (1 - q)\max_{j>0}\left(\frac{G(\bar{q}_j)}{G(\bar{q}_{j-1})}\right)$
- $\gamma^{**} = \max_{1\leq\pi\leq N}\gamma^*(q_\pi)$

Results

Lemma

At every round $t > t_1 = 2 + \log_{\gamma^{**}}((1 - \gamma^{**})(q_M - q_{M+1})/q_{M+1})$*:*

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$$\begin{cases} b_\pi > q_{M+1} & \pi < M+1, \\ b_\pi = q_\pi & \pi \geq M+1. \end{cases}$$

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Proof Approach.

"Dynamical system in γ^{**}." $\square$

Results

Convergence of the M Positions

Results

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- By the Lemma

Results

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- By the Lemma, we need only show that the M positions converge to the fixed point after round t_1.

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- By the Lemma, we need only show that the M positions $\boxed{\text{converge to the fixed point}}$ after round t_1.

Results

Convergence of the M Positions

- By the Lemma, we need only show that the M positions $\boxed{\text{stabilize}}$ after round t_1.

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    - If $\pi(P) = \{1, \ldots, M\}$, then we are done.

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- Set of stable positions: $P = \{p + 1, \ldots, M\}$
- Advertiser $\pi \notin \pi(P)$ with the lowest bid
    - Case 1: π targets position p
    - $\Rightarrow P' = P \cup \{p\}$ is stable

Results

Convergence of the M Positions

- Set of stable positions: $P = \{p+1, \ldots, M\}$
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  - Case 2: π targets position $\hat{p} > p$

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        - Depends upon the specific functional form of $\gamma_{\hat{p}}(q)$

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        - Depends upon the specific functional form of $\gamma_{\hat{p}}(q)$
        - (Significant divergence from Cary et al. (2008))

Results

Convergence of the M Positions

- Set of stable positions: $P = \{p+1, \ldots, M\}$
- Advertiser $\pi \notin \pi(P)$ with the lowest bid
  - Case 3: π targets position $\hat{p} < p$

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- Set of stable positions: $P = \{p+1, \ldots, M\}$
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  - $\Rightarrow$ P remains stable
    minimum bid of advertisers not in $\pi(P)$ increases

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Let
$\epsilon = \frac{G(\bar{q}_M)}{2G(\bar{q}_1)}(1-\gamma^{**})\min_{\phi\neq\phi'}|q_\phi - q_{\phi'}|\left(\prod_{j=1}^{M}(1-q_j)\right)$.

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If $b_{\pi_p} > q_\pi - \epsilon$*, then advertiser* π *prefers position* p *to any position* $\hat{p} < p$.

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If $b_{\pi_p} > q_\pi - \epsilon$*, then advertiser* π *prefers position* p *to any position* $\hat{p} < p$*.*

Proof Approach.

Very weak bound on the utility of position $\hat{p} < p$. □

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"Dynamical system in γ^{**}." □

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(Not dependent upon the form of $\gamma^*(q)$?) $\square$

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    after finitely many rounds, Case 1 or 2 must occur

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    - $\Rightarrow$ P remains stable
      minimum bid of advertisers not in $\pi(P)$ increases
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    - $\Rightarrow$ $\mathbb{QED}$

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Extension

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- Convergence in an asynchronous bidding model

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- $\boxed{\text{Efficient}}$ convergence in an asynchronous bidding model

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- Efficient convergence in an asynchronous bidding model with probability 1

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- Expected convergence time:

Extension

- Efficient convergence in an asynchronous bidding model with probability 1
- Expected convergence time:
    - explicit bound

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  - polynomial in N

Extension

- Efficient convergence in an asynchronous bidding model with probability 1
- Expected convergence time:
  - explicit bound
  - polynomial in N
  - of the same form as that of Cary et al. (2008)

Road Map

- Position Auctions (without consumer search)
  - Motivation
  - Static Model (Edelman, Ostrovsky, and Schwarz (2007))
  - Dynamic Model (Cary et al. (2008))
- Position Auctions with Consumer Search
  - Motivation
  - Static Model (Athey and Ellison (2008))
- Our Model
  - Convergence
- Generalizations?

Unwinding Our Method

Unwinding Our Method

- Our method $\approx$ Cary et al. (2008)'s method

Unwinding Our Method

- Our method $\approx$ Cary et al. (2008)'s method; its applicability is naïvely surprising.

Unwinding Our Method

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Three key steps:

Unwinding Our Method

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Three key steps:

1. restriction of the strategy space

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2. analysis of low-quality advertisers' behaviors
3. proof that the M positions stabilize

Unwinding Our Method

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Unwinding Our Method

1. restriction of the strategy space
   - “Restricted Balanced Bidding”

Unwinding Our Method

1. restriction of the strategy space
   - "Restricted Balanced Bidding"
   - ensures that the equilibrium in each round is unique

Unwinding Our Method

2. analysis of low-quality advertisers's behavior

Unwinding Our Method

2. analysis of low-quality advertisers's behavior
   - our first lemma

Unwinding Our Method

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   - low-quality advertisers should drop out efficiently

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   - low-quality advertisers should drop out efficiently
   - equilibrium bid monotonicity presumably unnecessary

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   - Cases 1–3

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   - we showed: top M advertisers do not bid over their equilibrium bids "too often"
   - may not hold in general—but is likely to hold when equilibrium is monotone

Conclusion

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Convergence should be demonstrable in dynamic position auction models with sufficiently well-behaved static equilibrium strategies.

Questions?