Clubs, Beliefs, and Entrapment

Scott Duke Kominers

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Question Why do people join clubs?

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Classical Answer

Because they have to.



Subtle Answer Because they have to.



Subtle Answer Because they think they have to.



Subtle Answer

Because they think they have to.





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Answer Because we have to.



Answer Because we have to.

• But the *first* tippers. . .



Answer Because we have to.

• But the *first* tippers wanted to.



Answer Because we have to.

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Answer Because we have to.

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Positive utility from early adoption



Answer Because we have to.

• But the *first* tippers wanted to.

Positive utility from early adoption + network effect



Answer Because we have to.

• But the *first* tippers wanted to.



Question

Answer

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Positive utility from early adoption + network effect = entrapment!

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Clubs, Beliefs, and Entrapment

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Answer

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Answer

Because we have to.

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Answer Because we have to.

• But the *first* VHS users...



Answer

Because we have to.

• But the *first* VHS users had to *hope* that VHS would catch on.



Answer

Because we have to.

• But the *first* VHS users had to *hope* that VHS would catch on.



Answer

Because we have to.

• But the *first* VHS users had to *hope* that VHS would catch on since Betamax was "better".



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Because we have to.

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Because we have to.

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Math Model

Notation

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Image: A matrix and a matrix

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• N > 1 players, indexed $i = 1, 2, \dots, N$

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JOIN STATUS QUO

Math Model

Notation

- N > 1 players, indexed $i = 1, 2, \dots, N$
 - Must (simultaneously) choose

$$\begin{array}{c|c} \text{JOIN} & \text{STATUS QUO} \\ J(i,n) & S(i,n) \end{array}$$

- N > 1 players, indexed $i = 1, 2, \dots, N$
 - Must (simultaneously) choose

JOIN | STATUS QUO
$$J(i, n)$$
 | $S(i, n)$

where n = number of others choosing JOIN

Math Model

Notation

• N > 1 players i = 1, 2, ..., N• $\begin{vmatrix} \text{JOIN} \\ J(i, n) \end{vmatrix}$ STATUS QUO $\begin{vmatrix} S(i, n) \end{vmatrix}$

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Our Assumptions

- N > 1 players $i = 1, 2, \dots, N$
- $\left| \begin{array}{c} \text{JOIN} & \text{STATUS QUO} \\ J(i,n) & S(i,n) \end{array} \right|$

Our Assumptions

• Players sorted in order of enthusiasm for JOIN:

$$\frac{\partial A}{\partial i} < 0, \quad \frac{\partial S}{\partial i} > 0$$

•
$$N > 1$$
 players $i = 1, 2, \dots, N$

• JOIN STATUS QUO
$$J(i, n)$$
 $S(i, n)$

Our Assumptions

• Players sorted in order of enthusiasm for JOIN:

$$\frac{\partial A}{\partial i} < 0, \quad \frac{\partial S}{\partial i} > 0$$

• Positive network effect:

$$\frac{\partial A}{\partial n} > 0, \quad \frac{\partial S}{\partial n} < 0$$

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$$N > 1$$
 players $i = 1, 2, \dots, N$

• JOIN STATUS QUO
$$J(i, n)$$
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Our Assumptions

 Functional forms (following Dixit (2003)):

Notation

• N > 1 players i = 1, 2, ..., N• $\begin{vmatrix} \text{JOIN} & \text{STATUS QUO} \\ J(i, n) & S(i, n) \end{vmatrix}$

Our Assumptions

 Functional forms (following Dixit (2003)):

$$J(i,n) = \beta + \gamma n - \delta i$$

$$S(i,n) = \sigma i - \tau n$$

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$$N > 1$$
 players $i = 1, 2, ..., N$
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• Early adoption positive:

$$\beta > \delta + \sigma$$

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• Strong network effect:

 $\gamma+\tau\geq\delta+\sigma$

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• $\begin{vmatrix} \text{JOIN} \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO
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•
$$N-1$$
 Join $\Rightarrow N$ Join:

$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Notation

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$$N > 1$$
 players $i = 1, 2, ..., N$
• $\begin{vmatrix} JOIN \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO $\sigma i - \tau n$

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Our Assumptions

• Common *ex ante* belief

Notation

•
$$N > 1$$
 players $i = 1, 2, ..., N$
• JOIN STATUS QUO
 $\beta + \gamma n - \delta i$ STATUS QUO
 $\sigma i - \tau n$

•
$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Our Assumptions

• Common *ex ante* belief:

 $\operatorname{Prob}(\operatorname{others} \operatorname{JOIN}) = q$

Notation

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$$N > 1$$
 players $i = 1, 2, ..., N$
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Our Assumptions

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Prob(others JOIN) = q

Key Condition

Notation

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$$N > 1$$
 players $i = 1, 2, ..., N$
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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Key Condition

• Player *i* JOINs \iff

Notation

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 players $i = 1, 2, ..., N$
• $\begin{vmatrix} JOIN \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO $\sigma i - \tau n$

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Key Condition

• Player i JOINS \iff

$$q(eta + \gamma(N-1) - \delta i) + (1-q)(eta - \delta i)$$

Notation

•
$$N > 1$$
 players $i = 1, 2, ..., N$
• JOIN STATUS QUO
 $\beta + \gamma n - \delta i$ STATUS QUO
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Key Condition

• Player i JOINs \iff

$$q(\beta + \gamma(N-1) - \delta i)$$

$$+ (1-q)(\beta - \delta i)$$

$$>$$

$$q(\sigma i - \tau(N-1))$$

$$+ (1-q)(\sigma i)$$

Notation

•
$$N > 1$$
 players $i = 1, 2, ..., N$
• JOIN STATUS QUO
 $\beta + \gamma n - \delta i$ STATUS QUO
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Our Assumptions

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

)

Key Condition

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Results

Notation

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Our Assumptions

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Prob(others JOIN) = q

Results

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At least
$$\left[N\left(\frac{\beta+q(N-1)(\gamma+\tau)}{\beta+(N-1)(\gamma+\tau)}\right) \right]$$

players JOIN!

Notation

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$$N > 1$$
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Results

- At least $\left\lfloor N\left(\frac{\beta+q(N-1)(\gamma+\tau)}{\beta+(N-1)(\gamma+\tau)}\right) \right\rfloor$ players JOIN!
- When β = 0 (no early adoption benefit), this is [Nq].

Notation

• N > 1 players i = 1, 2, ..., N• $\begin{vmatrix} \text{JOIN} \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO $\sigma i - \tau n \end{vmatrix}$

- $\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$
- Prob(others JOIN) = q

Results

- At least $\left\lfloor N\left(\frac{\beta+q(N-1)(\gamma+\tau)}{\beta+(N-1)(\gamma+\tau)}\right) \right\rfloor$ players JOIN!
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• *q* = 1

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Results

- At least $\left\lfloor N\left(\frac{\beta+q(N-1)(\gamma+\tau)}{\beta+(N-1)(\gamma+\tau)}\right) \right\rfloor$ players JOIN!
- When $\beta = 0$ (no early adoption benefit), this is $\lfloor Nq \rfloor$.

•
$$q = 1 \Rightarrow \text{all Join}$$

Notation

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 players $i = 1, 2, ..., N$
• $\begin{vmatrix} \text{JOIN} \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO
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Results

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$$q = 1 \Rightarrow \text{all JOIN}$$

• $q = 0$

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Results

- At least $\left\lfloor N\left(\frac{\beta+q(N-1)(\gamma+\tau)}{\beta+(N-1)(\gamma+\tau)}\right) \right\rfloor$
 - players JOIN!
- When β = 0 (no early adoption benefit), this is [Nq].

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$$q = 1 \Rightarrow \mathsf{all JOIN}$$

• $q = 0 \Rightarrow$ none JOIN

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convincing rumor

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- N > 1 players i = 1, 2, ..., N• $\begin{vmatrix} \text{JOIN} \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO $\sigma i - \tau n \end{vmatrix}$
- Our Assumptions

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

Prob(others JOIN) = q

Results

- $q = 1 \Rightarrow \text{all JOIN}$
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• N > 1 players i = 1, 2, ..., N• $\begin{vmatrix} \text{JOIN} \\ \beta + \gamma n - \delta i \end{vmatrix}$ STATUS QUO $\sigma i - \tau n$

+ network effect

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$$\frac{\beta + (N-1)(\gamma + \tau)}{N} > \delta + \sigma$$

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convincing rumor + network effect entrapment!

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- $q = 1 \Rightarrow \text{all Join}$
- $q = 0 \Rightarrow$ none JOIN

Notation

• N > 1 players i = 1, 2, ..., N• JOIN $\beta + \gamma n - \delta i$ STATUS QUO $\sigma i - \tau n$

+ convincing rumor + network effect entrapment!

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Acknowledgments

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Image: A matrix and a matrix

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• Prof. Avinash Dixit and Prof. Andrei Shleifer

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• Mrs. Susan Schwartz Wildstrom

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- Mrs. Susan Schwartz Wildstrom
- Harvard College PRISE

- Prof. Avinash Dixit and Prof. Andrei Shleifer
- Mrs. Susan Schwartz Wildstrom
- Harvard College PRISE
- Family

- Prof. Avinash Dixit and Prof. Andrei Shleifer
- Mrs. Susan Schwartz Wildstrom
- Harvard College PRISE
- Family, friends

- Prof. Avinash Dixit and Prof. Andrei Shleifer
- Mrs. Susan Schwartz Wildstrom
- Harvard College PRISE
- Family, friends, and you!

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- Family, friends, and you! (\mathbb{QED})